Lecture 10 - Distributed Element Matching Networks

Microwave Active Circuit Analysis and Design

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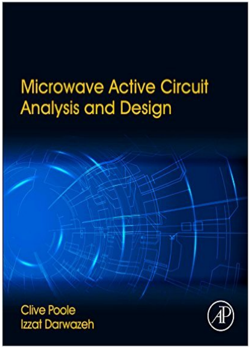

Intended Learning Outcomes

- Knowledge
  - Understand the advantages and disadvantages of distributed element matching, when compared to lumped element matching networks.
  - Be aware that distributed element matching networks can be designed either analytically (using a computer) or graphically using the Smith Chart.
  - Understand the principles behind stub matching networks and be aware that there are always two stub matching solutions to a given matching problem, which differ in terms of their bandwidth.
  - Understand the theory behind the quarter wave $(\lambda/4)$ transformer, its properties and applications.
  - Understand bandwidth performance of distributed element matching networks.
- Skills
  - Be able to design a single stub matching network to match an arbitrary load.
  - Be able to design a double stub matching network to match an arbitrary load.
  - Be able to design a quarter-wave transformer matching network to match an arbitrary load.
  - Be able to calculate the bandwidth of a single stub, double stub or quarter-wave transformer matching network.

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Impedance transformation with line sections

The simplest possible distributed matching network is just a single length of transmission line, of characteristic impedance Z_o, connected between load and source, as shown in figure 1.

The transmission line section has the effect of transforming any load impedance into some other impedance, determined by the length of the line section and the line characteristic impedance Z_o.

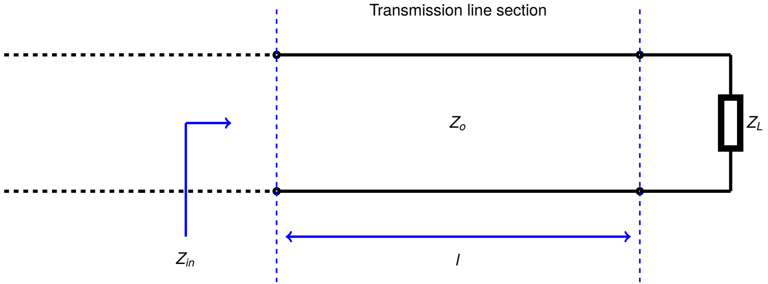

Figure 1 : Single line matching

Impedance transformation with line sections

The input impedance of the lossless transmission line section in figure 1, having arbitrary length, l, characteristic impedance Z_o, and terminated with the load Z_L, is given by equation (**??**):

$$Z_{in} = R_{in} + jX_{in} = Z_o \left(\frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)} \right) \quad (\textbf{??})$$

This can be expressed in admittance form as :

$$Y_{in} = G_{in} + jB_{in} = Y_o \left(\frac{Y_L + jY_o \tan(betal)}{Y_o + jY_L \tan(betal)} \right) \quad (1)$$

In order to match two impedances by using a length of transmission line connected between them we need to transform Y_L into the desired input admittance, Y_{in} by varying the quantities Y_o and l. The required relationship between Y_L and Y_{in} referred to above can be found by equating the real and imaginary parts of equation (1) and solving for l and Y_o. We equate the real parts of both sides of (1) and rearrange to obtain:

$$\tan(\beta l) = Y_o \left(\frac{G_{in} - G_L}{G_{in}B_L + G_L B_{in}} \right) \quad (2)$$

Impedance transformation with line sections

Equating imaginary parts of both sides of (1), substituting (2) and solving for Y_o gives:

$$Y_o = \sqrt{G_L G_{in} \left[1 + \frac{B_{in}^2 G_L - B_L^2 G_{in}}{G_L G_{in} (G_{in} - G_L)} \right]} \tag{3}$$

Since Y_o must be real for a practical line, the condition for single line matching to be realisable can be taken from (3) to be:

$$\frac{B_{in}^2 G_L - B_L^2 G_{in}}{G_L G_{in} (G_{in} - G_L)} > 1 \tag{4}$$

The realisability condition of equation (4) can be represented by circles on the Smith chart.

When condition (4) is not satisfied these circles have the advantage of revealing whether the addition of a parallel stub will move the load admittance into an area where single line matching is possible.

Due to this limitation the single line matching technique is rarely used in practice.

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Single Stub Matching

- The 'single stub' matching technique makes use of the fact that a length of transmission line terminated in either an open or short circuit will behave as a pure susceptance whose value depends on the nature of the termination (open or short) and the length of the line section. Such a section of line is called a *stub*.
- The single stub matching network actually consists of two parts : the stub itself, of length *l*, and a length of transmission line, length *d*, connected between the load and the point at which the stub is attached.
- A typical single stub matching network implemented in microstrip is illustrated schematically in figure 2.

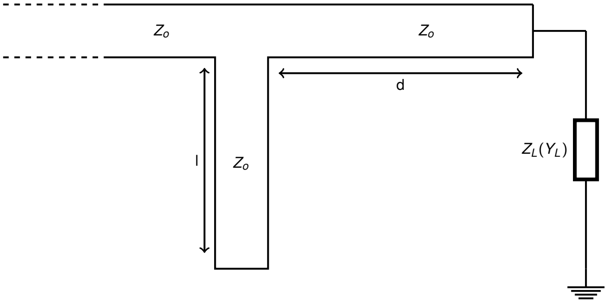

Figure 2 : Open circuit stub matching implemented in microstrip

Single Stub Matching

- Stubs are widely used in planar microwave circuits, such as microstrip and MMIC implementations as they are extremely easy to fabricate. In these media the stub is connected in parallel with the main line.
- Because a parallel connection is used, the design procedure is best carried out in terms of admittances.
- The open circuit stub configuration is shown in figure 3(a) and the short circuit stub configuration is shown in figure 3(b).
- The lines all have characteristic impedance Z_o (characteristic admittance Y_o).

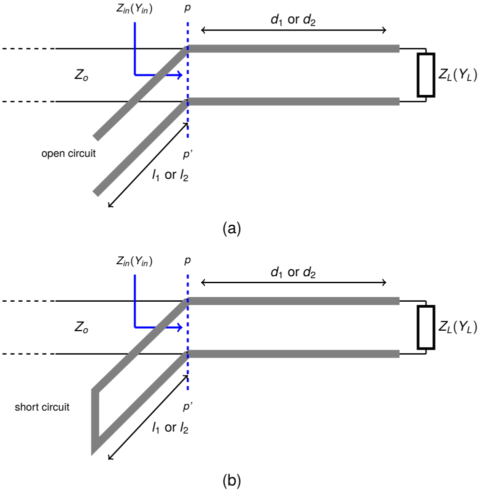

Figure 3 : Generic single stub matching networks : (a) Open circuit stub, (b) Short circuit stub

Single stub matching : Analytical approach

We start by defining the input admittance of the line at the plane pp' in figure 3(a) or figure 3(b), at a distance 'd' from the load as Y_{in}. Applying equation (1), and normalising by dividing by Y_o, gives us the normalised admittance, y_{in}, looking into the line at the plane pp' as:

$$y_{in} = \frac{y_L + j\tan(\beta d)}{1 + jy_L\tan(\beta d)} = \frac{y_L + jt}{1 + jy_L t} \tag{5}$$

where:

$$t = \tan(\beta d) \tag{6}$$

$$\beta = \frac{2\pi}{\lambda} \tag{7}$$

In general y_L will be complex, i.e.:

$$y_L = g_L + jb_L \tag{8}$$

so we can rewrite (5) as:

$$y_{in} = \frac{g_L + j(b_L + t)}{(1 - b_L t) + jg_L t} \tag{9}$$

Single stub matching : Analytical approach

Realising the denominator of (9) gives us the real and imaginary parts of y_{in} as follows:

$$g_{in} = \frac{g_L(1+t^2)}{(1-b_L t)^2 + g_L^2 t^2} \tag{10}$$

$$b_{in} = \frac{b_L(1-t^2) + t(1-b_L^2-g_L^2)}{(1-b_L t)^2 + g_L^2 t^2} \tag{11}$$

The essential idea behind single stub matching is that at some point on the line normalised susceptance looking into the line, g_{in}, will be unity. At this point, a parallel stub can be added to exactly cancel the line susceptance, b_{in}, resulting in a perfect match, i.e. $y_{in} = 1 + j0$. The distance between the load and this point, *pp'* in figure 3(a) or figure 3(b), where the stub is to be attached, can be calculated by setting g_{in} equal to unity in equation (10), which results in the following quadratic in t:

$$t^2(g_L^2 + b_L^2 - g_L) - 2b_L t + (1-g_L) = 0 \tag{12}$$

This can be solved for t as follows :

$$t = \frac{b_L \pm \sqrt{g_L((1-g_L)^2 + b_L^2)}}{g_L^2 + b_L^2 - g_L} \tag{13}$$

Single stub matching : Analytical approach

From (13) we can determine the value of d, as a fraction of a wavelength, as follows :

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi}\tan^{-1}(t) & \text{if } t \geq 0 \\ \frac{1}{2\pi}(\tan^{-1}(t) + \pi) & \text{if } t < 0 \end{cases} \quad (14)$$

It can easily be shown that, in the special case of purely resistive loads (i.e. $b_L = 0$) there is a single solution to (12), namely :

$$t = \sqrt{\frac{1}{g_L}} \quad (15)$$

In the more general case of complex loads, we can see from (13) and (14) that there are two values of line length, d, that satisfy the condition $g_{in} = 1$. Let us refer to these as d_1 and d_2.

For each value of d determined from equation (13) and (14), the susceptance of the stub required to cancel the line susceptance can be found by entering the value, d_1 or d_2 (i.e. the values corresponding t_1 or t_2), into equation (11). The stub susceptance is simply the same magnitude but opposite polarity of the b_{in} value thus calculated.

Single stub matching : Analytical approach

The length of the stub needed to realise the susceptance $b_{stub} = -b_{in}$, is given by the familiar equations for normalised susceptance of a lossless line terminated in open circuit and short circuit respectively (which can be easily derived by setting $y_L = 0$ or $y_L = \infty$ in (9)):

$$b_{open} = \tan \beta l_{open} \tag{16}$$

$$b_{short} = -\cot \beta l_{short} \tag{17}$$

Where l_{open} and l_{short} refer to the lengths of the stubs in figure 3(a) and figure 3(b), respectively. Note that, since there are two values of d that satisfy (13), there will be a different value of either open circuit or short circuit stub length corresponding to each value of d. In other words, there will be four possible matching network solutions, as follows :

1. d_1 with an open circuit stub.
2. d_1 with a short circuit stub.
3. d_2 with an open circuit stub.
4. d_2 with a short circuit stub.

If any of the equations above result in negative electrical lengths, or an electrical length that is impractically short, we can simply add $\pm(n\lambda)/2$ to get an equivalent positive electrical length.

Single stub matching : Graphical approach

The procedure can be summarised in 4 steps as follows :

Step 1: Plot the normalized load impedance on the Smith chart

Step 2: Draw the contant VSWR circle and determine the normalised load admittance

Step 3: Move around the constant VSWR circle toward the source until you cross the unit conductance circle. At this point, the normalised admittance looking into the line, y_{in}, is 1 + jb (or 1-jb, depending on which intersection with the unit conductance circle is chosen).

Step 4: Add in a shunt stub at this point with susceptance of equal magnitude and opposite sign to $\pm b$. This cancels the transformed load susceptance at the attachment point, resulting in a perfect match : $y_{in} = 1 + j0$

The procedure is best illustrated by an example, in the following slides.

Single stub matching : Graphical example

As an illustrative example, we will consider the matching of a load $Z_L = 25 - j50\Omega$ to a 50Ω transmission line system, using the graphical method, as follows:

- Normalise the load : $z_L = (25 - j50)/50 = 0.5 - j$ and plot this point on the Smith Chart (indicated as 'z_L' in figure 4).
- Draw the constant VSWR circle through this point and thereby determine the normalised load admittance as $y_L = 0.4 + j0.8$, which is 180^o around the VSWR circle (indicated as 'y_L' in figure 4). Note that from this point on, all points on the Smith Chart are to be interpreted as *normalised admittances*.

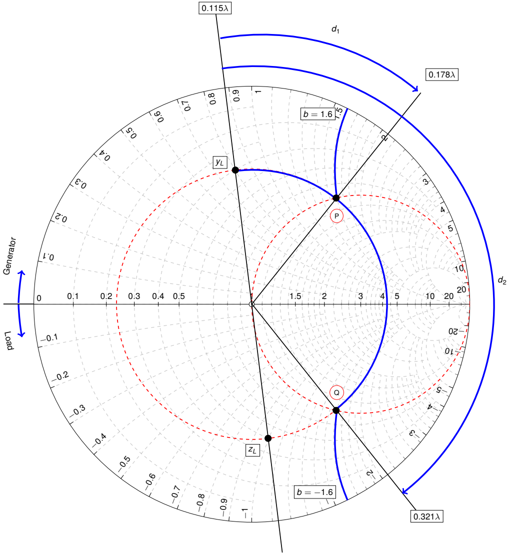

Figure 4 : Single stub matching: graphical method

Single stub matching : Graphical example

- Starting at y_L, we move around the constant VSWR circle clockwise towards the generator until we cross the unit conductance circle at point 'P' in figure 4. The normalised admittance looking into the line at point 'P' is now 1+jb. Note that if we keep going clockwise around the constant VSWR circle we will cross the unit conductance circle again at point 'Q'. The normalised admittance looking into the line at point 'Q' is 1-jb. The lengths of line, from the load to the point of stub attachment, in each of these cases is d_1 and d_2 respectively. In our example, we can determine the line lengths, in terms of guided wavelength, by reading off the 'wavelengths towards generator' scale on the outer boundary of the Smith chart. This gives us the following :

$$d_1 = (0.178 - 0.115)\lambda = 0.063\lambda$$
$$d_2 = (0.321 - 0.115)\lambda = 0.206\lambda$$

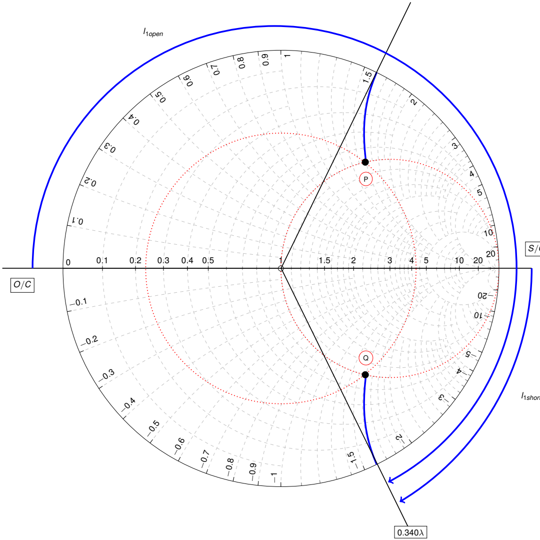

Figure 5 : Single stub matching: determination of stub lengths for point 'P'

Single stub matching : Graphical example

- We now need to determine the susceptance of the shunt stub to be attached at either point 'P' or point 'Q'. This should be equal in magnitude and opposite in sign to $\pm b$, as appropriate. The addition of the stub will cancel the transformed load susceptance at the attachment point, resulting in a perfect match at that point, i.e. $y_{in} = 1 + j0$. We determine the input susceptance of the line at the point of stub attachment by finding the constant susceptance circle which intersects the constant conductance circle at points 'P' and 'Q' in figure 6. The values in our example are b=1.6 and b=-1.6 respectively.

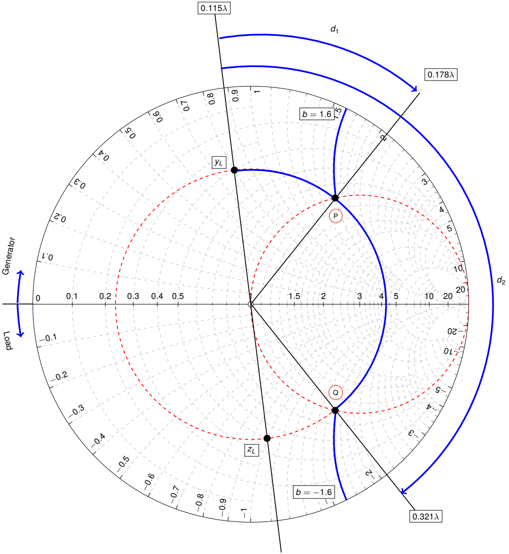

Figure 6 : Single stub matching: graphical method

Single stub matching : Graphical example

- Now that we know the input susceptance of the line where the stub is to be attached (points 'P' or 'Q'), we can use the Smith Chart to determine the length of the short or open circuit stub required. This procedure is illustrated for point 'P' in figure 7 and for point 'Q' in figure 8.
- Note that the stub susceptance is of opposite sign to the line input susceptance. The length of the stub is found by moving clockwise around the chart starting at the open circuit ($y = 0$) or short circuit ($y = \infty$) points (labeled 'O/C' and 'S/C' respectively in figure 7 and figure 8) until the desired location on the periphery of the Smith chart is reached.
- This location is found by drawing a radial through the point where the constant reactance circle intersects the boundary of the Smith chart. We can then read off the value on the 'wavelengths towards generator' scale.

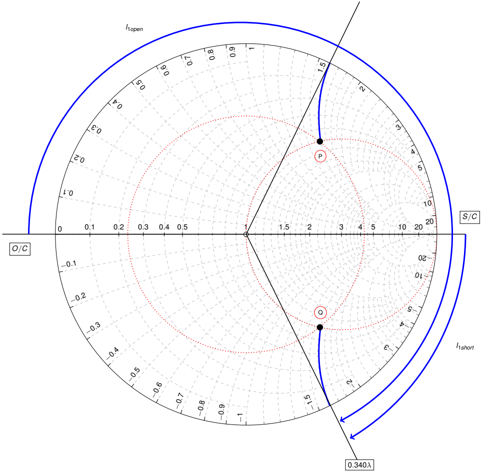

Figure 7 : Single stub matching: determination of stub lengths for point 'P'

Single stub matching : Graphical example

The respective stub lengths are $l_1 open$ and $l_1 short$ in the case of point 'P' (figure 7) and $l_2 open$ and $l_2 short$ in the case of point 'Q' (figure 8). In our example these values have been determined with reference to figure 7 and figure 8 as follows :

$$l_{1open} = 0.340\lambda$$
$$l_{1short} = (0.340 - 0.25)\lambda = 0.09\lambda$$
$$l_{2open} = 0.160\lambda$$
$$l_{2short} = (0.160 + 0.25)\lambda = 0.410\lambda$$

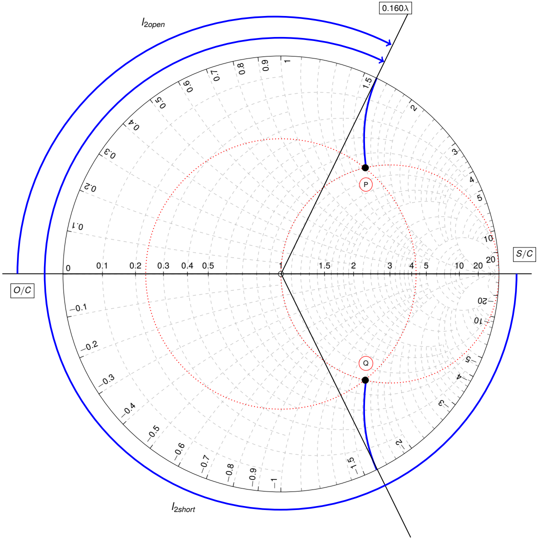

Figure 8 : Single stub matching: determination of stub lengths for point 'Q'

Single stub matching example : solutions

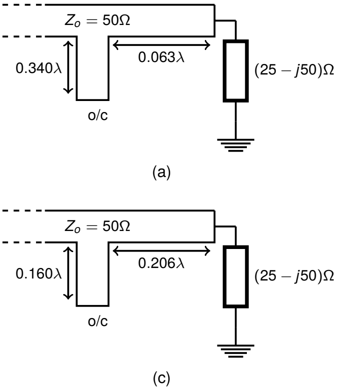

Figure 9 : Four possible microstrip stub matching networks for the load $25 - j50\Omega$: (a) Point 'P' - open circuit stub, (b) Point 'P' - short circuit stub, (c) Point 'Q' - open circuit stub, (d) Point 'Q' - short circuit stub

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Double Stub Matching

- One limiting characteristic of single stub matching is that the location of the matching stub is a function of the load impedance. This may not be an issue in most cases, but in some practical circumstances there may be constraints on the physical positioning of the stub.
- In such cases we can use an alternative matching technique called *Double stub matching*, which employs two stubs, spaced a fixed distance apart.
- The advantage of the double stub matching approach is that, although the distance between the two stubs is generally kept constant, the first stub may be placed at any distance from the load.
- For this reason, the double stub technique is often used to implement variable tuners, where the match can be adjusted by varying only the stub lengths, with the stub locations on the main line remaining fixed.

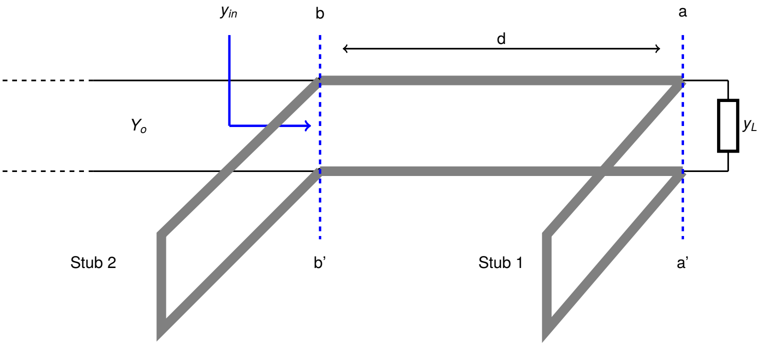

Figure 10 : Double stub matching, first stub at load

Double stub matching : Analytical approach

Let us define $y_{bb'}$ as the normalised input admittance looking into the line at the plane bb' in the absence of Stub 2. If the susceptance of Stub 1 has been correctly chosen (using a method we will explain shortly) then $y_{bb'}$ can be written as :

$$y_{bb'} = I + jb_{bb'}$$

or

$$y_{bb'} = I - jb_{bb'}$$

Where $\pm b_{bb'}$ is the residual susceptance of the line. We now add Stub 2 to cancel this residual line susceptance. The line to the left of Stub 2 will then be matched to the load since,

$$y_{in} = y_{bb'} + jb_{Stub2}$$

or

$$y_{in} = y_{bb'} - jb_{Stub2}$$

Where b_{Stub2} is equal and opposite in sign to $b_{bb'}$. We therefore have a perfect match at bb', i.e. $y_{in} = 1$.

As with single stub matching, we will outline both analytical and graphical design approaches.

Double stub matching : Analytical approach

The normalised admittance at the load in figure 10, with Stub 1 attached is :

$$y_{aa'} = g_L + j(b_L + b_{Stub1})$$

Where b_L is the load susceptance and b_{Stub1} is the susceptance of Stub 1. After this impedance has been transformed through the length of line, d, and prior to the attachment of Stub 2, the normalised admittance at point bb' in figure 10 is :

$$y_{bb'} = \frac{g_L + j(b_L + b_{Stub1} + t)}{1 + jt(g_L + j(b_L + b_{Stub1}))} \tag{18}$$

where, again:

$$t = \tan(\beta d) \tag{19}$$

and:

$$\beta = \frac{2\pi}{\lambda} \tag{20}$$

Double stub matching : Analytical approach

We know that the normalised conductance at point bb' must be unity, since this is where we will attach Stub 2 (which can only add susceptance) to achieve a perfect match. We therefore set the real part of equation (18) equal to 1, which gives us the following relationship between g_L, b_L and t :

$$g_L^2 - g_L \frac{1+t^2}{t^2} + \frac{(1-t(b_L+b_{Stub1}))^2}{t^2} = 0 \tag{21}$$

Re-arranging (21) yields the requisite value of b_{Stub1} as :

$$b_{Stub1} = -b_L + \frac{1 \pm \sqrt{g_L(1+t^2) - g_L^2 t^2}}{t} \tag{22}$$

Double stub matching : Analytical approach

The residual line susceptance at the point bb', with Stub 1 attached, is given by the imaginary part of (18) :

$$b_{bb'} = \frac{(1 - t(b_L + b_{Stub1}))(b_L + b_{Stub1} + t) - g_L^2 t}{(1 - t(b_L + b_{Stub1}))^2 + t^2 g_L^2} \tag{23}$$

Now, substituting the value of b_{Stub1} given by (22) into (23) gives us the residual line susceptance at bb' solely in terms of g_L and t :

$$b_{bb'} = \frac{\mp\sqrt{g_L(1 + t^2) - g_L^2 t^2} - g_L}{g_L t} \tag{24}$$

The susceptance of Stub 2 is chosen to cancel the residual line susceptance at bb', therefore :

$$b_{Stub2} = -b_{bb'}$$

Double stub matching : Analytical approach

We are now in a position to define b_{Stub2} solely in terms of g_L and t as follows :

$$b_{Stub2} = -\left(\frac{\mp\sqrt{g_L(1+t^2) - g_L^2 t^2} - g_L}{g_L t}\right) \tag{25}$$

Where the $\mp$ in (25) is paired with the $\pm$ in (22).
Once we have determined the required stub susceptances from (22) and (25), the electrical length of the stubs can be calculated for either short or open circuited stub by applying the familiar relationships (16) and (17).

Double Stub Matching Analytical Example

You need to match load consisting of a 4nH inductor which has a series internal resistance of 19.2 Ω to a 50 Ω lossless co-axial transmission line by means of a double stub matching network, consisting of two short circuit stubs, spaced 0.375λ apart. The stub nearest to the load is 0.1λ away from it. Determine the possible combinations of stub lengths which are required to match the load to the line at the operating frequency of 1835 MHz. You may assume all line sections and stubs are 50 Ω.
The load impedance is first calculated :

$$
\begin{aligned}
Z_L &= 19.2 - j(2\pi \times 1.835 \times 10^9 \times 4 \times 10^{-9}) \\
Z_L &= 19.2 + j46.17\Omega
\end{aligned}
$$

The normalised load impedance and admittance are therefore:

$$
Z_L = \frac{(19.2 + j46.17)}{50} = 0.384 + j0.923
$$

$$
\begin{aligned}
y_L &= \frac{1}{(0.384 + j0.923)} \\
&= \frac{0.384 - j0.923}{0.9994} \\
&= 0.384 - j0.923
\end{aligned}
$$

Double Stub Matching Analytical Example

Stub 1 is attached at a distance $l_x = 0.1\lambda$ from the load, so we need to work out the admittance $y_{aa'}$ at this point of attachment. We can employ equations (18) and (19) which give us the conductance and susceptance of a load, y_L, translated a distance, l_x, towards the generator. In this case $t = \tan(0.2\pi) = 0.727$, therefore :

$$g_{aa'} = \frac{0.384(1 + 0.727^2)}{(1 + 0.923 \times 0.727)^2 + 0.384^2 \times 0.727^2}$$

$$g_{aa'} = \frac{0.587}{2.870}$$

$$g_{aa'} = \boxed{0.204}$$

and

$$b_{aa'} = \frac{-0.923(1 - 0.727^2) + 0.727(1 - (-0.923)^2 - 0.384^2)}{(1 + 0.923 \times 0.727)^2 + 0.384^2 \times 0.727^2}$$

$$b_{aa'} = \frac{-0.435}{2.870}$$

$$b_{aa'} = \boxed{-0.154}$$

Double Stub Matching Analytical Example

Using this value of $y_{aa'} = 0.204 - j0.154$ as the translated load, we now calculate the required value of Stub 1 using equation (22), this time using $t = tan(0.375 \times 2\pi) = -1$:

$$
\begin{aligned}
b_{Stub1} =& 0.154 + \frac{1 \pm \sqrt{0.204(1 + (-1)^2) - 0.204^2 \times (-1)^2}}{-1} \\
b_{Stub1} =& -0.846 \pm (-0.606)
\end{aligned}
$$

We therefore have two solutions for Stub 1, namely :

$$b_{Stub1} = \boxed{-1.452}$$

and

$$b_{Stub1} = \boxed{-0.240}$$

Double Stub Matching Analytical Example

We calculate the admittance of Stub 2 by applying equation (25) :

$$b_{Stub2} = -\left(\frac{\mp\sqrt{2 \times 0.204 - 0.204^2} - 0.204}{-0.204} \right)$$

$$b_{Stub2} = -\left(\frac{\mp 0.605 - 0.204}{-0.204} \right)$$

We therefore have two solutions for Stub 2, namely :

$$b_{Stub2} = \boxed{1.967}$$

and

$$b_{Stub2} = \boxed{-3.967}$$

Double Stub Matching Analytical Example

We can now apply equations (16) and (17) to determine the length of short circuit and open circuit Stub 1 and 2 for both solutions as summarised in table 1.

Table 1 : Double stub matching: analytical solutions

	Normalised susceptance	Length (s/c stub)	Length (o/c stub)
Solution 1 :	Stub 1 = -1.452 Stub 2 = -3.967	Stub 1 = 0.096λ Stub 2 = 0.039λ	Stub 1 = 0.346λ Stub 2 = 0.289λ
Solution 2 :	Stub 1 = -0.240 Stub 2 = 1.967	Stub 1 = 0.212λ Stub 2 = 0.425λ	Stub 1 = 0.462λ Stub 2 = 0.175λ

Double stub matching : Graphical approach

- Double stub matching networks can also be designed graphically using the Smith Chart.
- Since we are adding elements in parallel so we will be using the Smith Chart is being used in admittance mode.

Consider the arrangement shown in figure 11. The total admittance at the point aa' is

$$y_{aa'} = y_L + jb_{Stub1}$$

where y_L is the admittance of the load and b_{Stub1} is the susceptance of Stub 1.

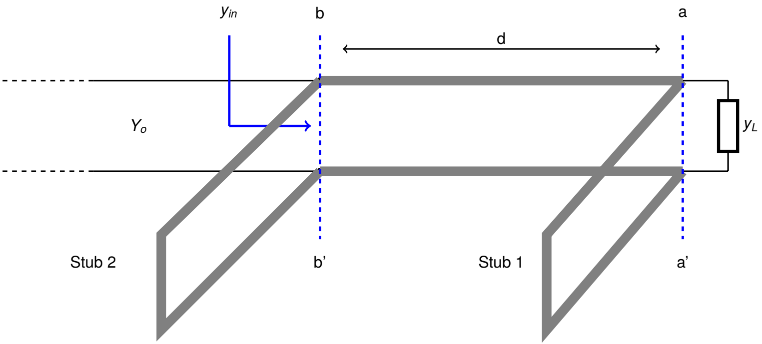

Figure 11 : Double stub matching, first stub at load

Double stub matching : Graphical approach

- Since stub 2 can only contribute susceptance, $y_{bb'}$ must be some point on the unit conductance circle on the Smith Chart (circle 1 in figure 13). We therefore deduce that $y_{aa'}$ must lie on a circle of equal radius but having its centre rotated d wavelength towards the load.
- This principle can be illustrated by considering figure 12 where we have chosen d to be $\lambda/4$. Picking an arbitrary point, A, which lies on the unit conductance circle, we see that the effect of adding the line d is to rotate this point around the constant VSWR circle through A, 0.25λ towards the load to a point A'.
- Another point B on the unit conductance circle is similarly rotated 0.25λ to point B'. The same procedure can be carried out for all the other points C to J in figure 12.

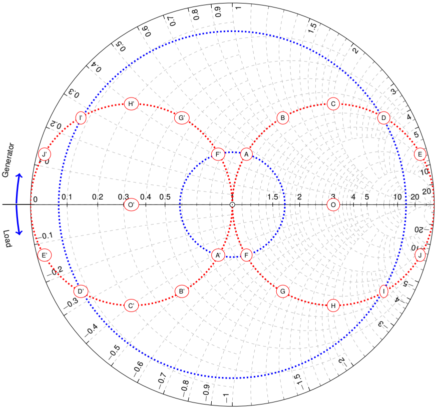

Figure 12 : The principle behind double stub matching

Double stub matching : Graphical approach

- If the locus is then drawn through all the primed points in figure 12, it is found to be a circle of the same radius as the unit conductance circle whose centre O' has been rotated 0.25λ towards the load from O.
- A similar transformation exists for any other value of *d*. in each case the locus of points on the unit conductance circle maps into a circle of the same radius, whose centre lies *d* wavelengths counter-clockwise towards the load from the centre of the unit conductance circle.
- Having established this principle, the method of determining the stub lengths using the Smith Chart may be illustrated with reference to figure 13.

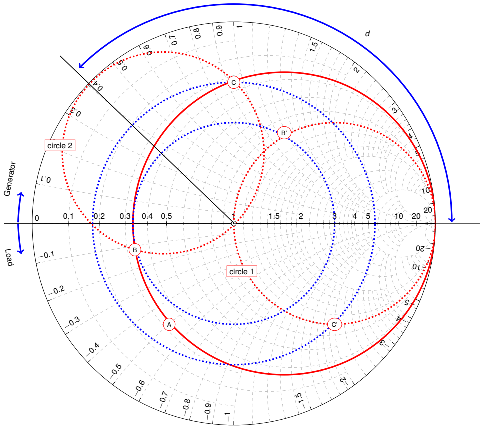

Figure 13 : Double stub matching : graphical method

Double stub matching : Graphical approach

- In figure 14, circle 2 represents the unit conductance circle transformed through some distance *d* wavelengths towards the load by means of the fixed line section, *d*, in figure 10.
- Let us pick a load admittance represented by the point A in figure 13. The constant conductance circle (solid circle) through point A cuts circle 2 at two locations, B and C. Hence, to achieve a match, we need to transform A to point B or C by choosing a value of Stub 1 which adds the correct susceptance to move point A in the right direction along the constant conductance circle to either point B or point C.
- The first stub, therefore, must present a normalised susceptance equal to the difference between the susceptance at point A and the susceptance at points B or C.

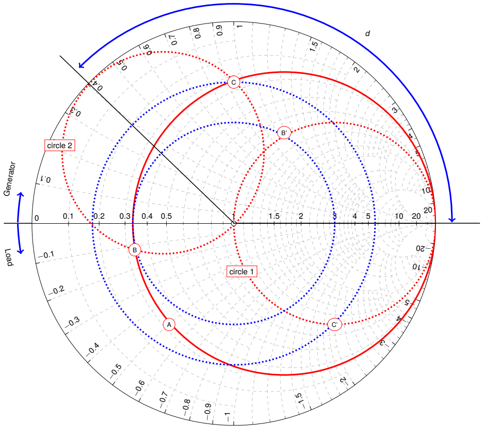

Figure 14 : Double stub matching : graphical method

Double stub matching : Graphical approach

- We determine the susceptance values by looking at which constant susceptance circles the points A,B and C lie on and subtracting one susceptance value from another. With the addition of Stub 1 with a value thus calculated, the input admittance at point B or C becomes $y_{aa'}$.
- The fixed line section, of length d, now transforms $y_{aa'}$ to $y_{bb'}$. The admittance at B or C when translated d wavelengths towards the generator will be given by points B' or C', respectively, on the unit conductance circle. This means that the admittance at B' is $y_{bb'} = 1 + jb_{B'}$ and the admittance at C' is $y_{bb'} = 1 + jb_{C'}$.
- The second stub therefore must present a normalised susceptance equal and opposite to the susceptance value at B' and C', these values being determined by looking at the susceptance scale on the Smith Chart.

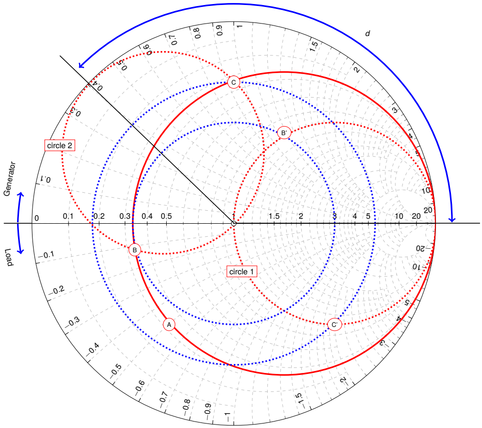

Figure 15 : Double stub matching : graphical method

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Double Stub Matching : forbidden regions

There are limitations on the load admittance that can be matched with the configuration shown in figure 10, as we will now show. Equation (21) is a quadratic in g_L which has the following solution :

$$g_L = \frac{1+t^2}{t^2}\left[1 \pm \sqrt{1 - \frac{4t^2(1-t(b_L+b_{Stub1}))^2}{(1+t^2)^2}}\right] \tag{26}$$

We note that the term inside the square root in equation (26) is of the form (1 - x). In order to meet the requirement that g_L should be a real number, the value of x must lie between 0 and 1. In the case of equation (26), this leads to the following boundaries on the value of g_L :

$$0 \leq g_L \leq \frac{1+t^2}{t^2} \tag{27}$$

With reference to standard trigonometric identities, (27) can be restated as :

$$0 \leq g_L \leq \frac{1}{\sin^2 \beta d} \tag{28}$$

Double Stub Matching : forbidden regions

- The limiting conditions set by (28) define a *forbidden region* on the Smith Chart within which loads are unmatchable with a double stub tuner. The forbidden region is bounded by a constant conductance circle whose value depends solely on the electrical stub separation, d/λ.
- Let's say, for example, we set $d = \lambda/8$. Then $\beta d = \pi/4$ and $\sin^2 \beta d = 0.5$. This means that the forbidden region will be bounded by the $g = 2$ circle, as shown in figure 16.
- In other words, only constant conductance circles lying outside the forbidden region in figure 16, can intersect with the translated g=1 circle.

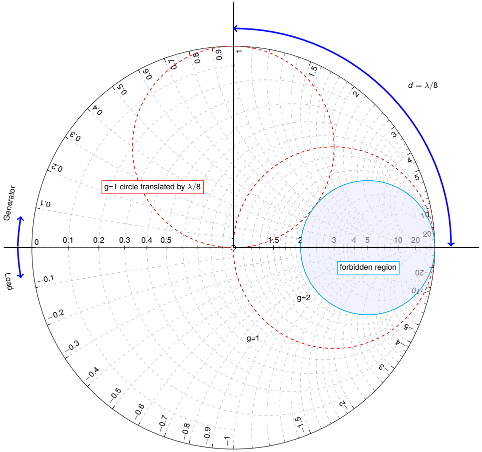
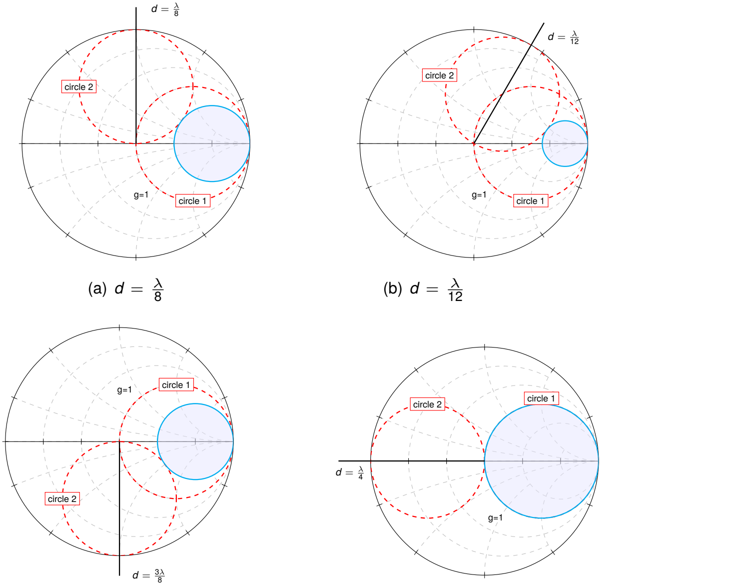


Figure 16 : Double stub matching: forbidden region for d=$\lambda/8$

Double Stub Matching : forbidden regions



(a) $d = \frac{\lambda}{8}$

(b) $d = \frac{\lambda}{12}$

(c) $d = \frac{3\lambda}{8}$

(d) $d = \frac{\lambda}{4}$

Double Stub Matching

If g_L lies within the forbidden region (i.e. it is greater than the value set by (28)) then we can still use the double stub technique with the first stub located a minimum distance, l_x, away from the load towards the generator, as shown in figure 17.

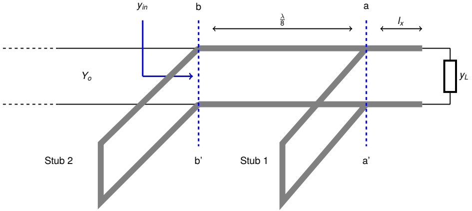


Figure 17 : Double stub matching, first stub located l_x from the load

Double Stub Matching

- The added line length has the effect of transforming the load admittance so that the transformed value of g_L is reduced to a value below the limit set by (28). This procedure is illustrated in figure 18, where point A represents the normalised admittance of the load and points A_1, A_2 and A_3 represent the admittance of the load transferred to a point on the line towards the generator at a distance $l_x = l_1, l_2$, or l_3 respectively from the load.
- Referring to figure 18, it can be seen that the constant conductance circle through point A, representing the load admittance, never intersects the unit conductance circle transferred towards the load by $\lambda/8$ (circle 2). Hence, this load cannot be matched using this particular double stub tuner (i.e. with $d = \lambda/8$) when the first stub is placed at the load.

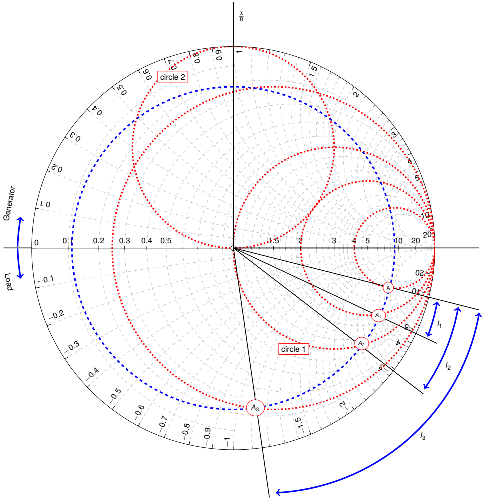


Figure 18 : Double stub matching : location of Stub 1 with respect to the load

Double Stub Matching

- To achieve a match, the load conductance must be reduced to such a value that the constant conductance circle through the new load point on the Smith Chart intersects circle 2. This can be achieved by shifting the point of application of the first stub towards the generator, in other words by adding the length of line l_x in figure 17.
- The transformed value of load conductance is gradually reduced as the distance l_x is increased. This is equivalent to moving clockwise around the constant VSWR circle, starting at A, in figure 18 and moving through points A_1, A_2, A_3, etc. It can be seen from figure 18 that if the point of application of the first stub (points aa' in figure 17) is at a distance equal to or greater than l_1, an effective match can be achieved, as the constant conductance circles through A_1, A_2, A_3, etc. all intersect circle 2.
- We conclude, therefore, that for double stub matching to be effective, Stub 1 must be connected at a minimum distance l_1 from the termination, where the value of l_1 depends on the value of the load conductance.
- Alternatively, by keeping the Stub 1 connected right at the load, the same result can be achieved by altering the distance between the stubs. If Stub 1 is connected at the load, then an effective match can be achieved by altering the distance between the stubs until the circle 2 cuts the constant conductance circle through the load admittance point.

Double Stub Matching Graphical Example

Design a double stub matching network to match a load of $\Gamma_L = 0.667\angle 90^o$ to the a 50 Ω lossless line. The two stubs are spaced 0.375λ apart. The stub nearest to the load is 0.1λ away from it. Carry out two designs: one using open circuit and one using short circuit stubs.

- We start by drawing the unit conductance circle (circle 1) and the same circle rotated through $\frac{3}{8}\lambda$ towards the load (circle 2) in figure 19.
- We then locate the load reflection coefficient = $0.667\angle 90^o$ as the impedance point 'A' on the Smith Chart, and draw the constant VSWR (=5) circle through this point. The VSWR=5 circle is designated as circle 3 in figure 19.
- We obtain the load admittance as point B by rotating the point A around circle 3 through 180^o in either direction. From this point onwards, all coordinates on the Smith Chart represent normalised admittance ($g + jb$).

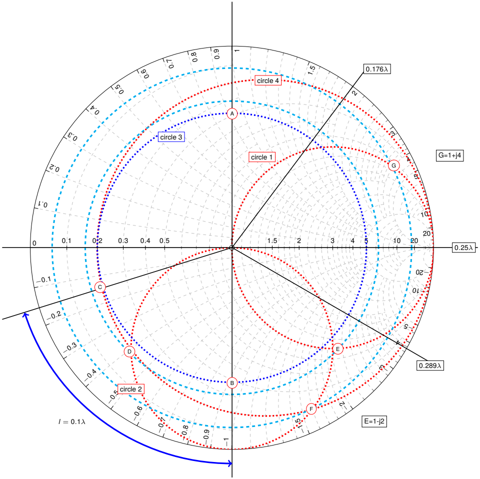


Figure 19 : Double stub matching, Example 1

Double Stub Matching Graphical Example

We are told that the distance to Stub 1 is 0.1λ from the load. We therefore locate point C by rotating the load admittance 0.1λ towards the generator in figure 19. We note that point C lies on the g=0.2 constant conductance circle (circle 4), which means that we can move back and forth along this circle by adjusting the susceptance of Stub 1.

We see that the g=0.2 constant conductance circle (circle 4) intersects the translated unit conductance circle (circle 2) at points D and F. Since there are two points of intersection, there must be two values of Stub 1 that can provide a match. The normalised susceptance corresponding to point D is found by looking at which constant susceptance circles points C and D lie on and subtracting one from the other. This gives us :

$$b_{Stub1D} = (-0.40) - (-0.15) = -0.25 \tag{29}$$

We locate the -0.25 constant susceptance circle on the perimeter of the Smith Chart and read off the value of 0.461λ on the 'wavelengths toward generator' scale. For a short circuit stub, we trace the stub length starting at the $y = \infty$ point on the Smith Chart. The electrical length of the short circuit Stub 1 for point D is therefore :

$$l_{Stub1Ds} = 0.461 - 0.25 = 0.211\lambda \tag{30}$$

The open circuit Stub 1 for point D is $\pm 0.25\lambda$ away (we obviously chose $\pm 0.25\lambda$ to give us a positive electrical length) :

$$l_{Stub1Do} = 0.211 + 0.25 = 0.461\lambda \tag{31}$$

Double Stub Matching Graphical Example

We apply the same approach to determine the susceptance of Stub 1 for point F :

$$b_{Stub1F} = (-1.6) - (-0.15) = -1.45 \tag{32}$$

Locating the point representing susceptance $b = -1.45$ on the 'wavelengths toward generator scale' gives us the following stub lengths:

$$\begin{aligned} l_{Stub1Fs} &= 0.346 - 0.25 = 0.096\lambda \\ l_{Stub1Fo} &= 0.096 + 0.25 = 0.346\lambda \end{aligned}$$

The length of line separating the two stubs ($d = 0.375\lambda$) has the effect of translating point D and F into the corresponding points E and G on the unit conductance circle in figure 19. The susceptance of Stub 2 can be determined by looking at which constant susceptance circle these two points lie on, and remembering that the susceptance of Stub 2 must have equal magnitude but opposite sign in order to cancel the residual line susceptance. Once again, since there are two intersections with the unit conductance circle, we have two possible values of Stub 2 that can provide a match. Note that point D is paired with point E and point F is paired with point G, so there are actually only two possible solutions consisting of the stub pairs D,E and F,G.

Double Stub Matching Graphical Example

For points E and G we can read off the residual line susceptances as :

$$b_E = -2.00$$
$$b_G = 4.00$$

Locating the opposite polarity susceptance points on the 'wavelengths towards generator' scale on the outer boundary of the Smith chart gives the following electrical lengths for Stub 2 using short circuit stubs :

$$l_{Stub2Gs} = 0.289 - 0.25 = 0.039\lambda$$
$$l_{Stub2Es} = 0.176 + 0.25 = 0.426\lambda$$

The equivalent open circuit stubs are $\pm 0.25\lambda$ in length, therefore :

$$l_{Stub2Go} = 0.039 + 0.25 = 0.289\lambda$$
$$l_{Stub2Eo} = 0.426 - 0.25 = 0.176\lambda$$

Double Stub Matching Graphical Example

So, we have four possible double stub matching network solutions, which are summarised in table 2.

Table 2 : Double stub matching: graphical solutions

	Normalised susceptance	Length (s/c stub)	Length (o/c stub)
Solution 1 : (D,E)	Stub 1 = -0.25 Stub 2 = 2.00	Stub 1 = 0.211λ Stub 2 = 0.426λ	Stub 1 = 0.461λ Stub 2 = 0.176λ
Solution 2 : (F,G)	Stub 1 = -1.45 Stub 2 = -4.00	Stub 1 = 0.096λ Stub 2 = 0.039λ	Stub 1 = 0.346λ Stub 2 = 0.289λ

Notwithstanding the arbitrary assignment of "Solution 1" and "Solution 2" categories, and rounding errors, the reader can see that these results are basically the same as those obtained by numerical calculation in example **??**.

Double Stub Matching Graphical Example

For clarity, a schematic representation of the two double stub matching network solutions implemented in microstrip implementation is shown in figure 20 :

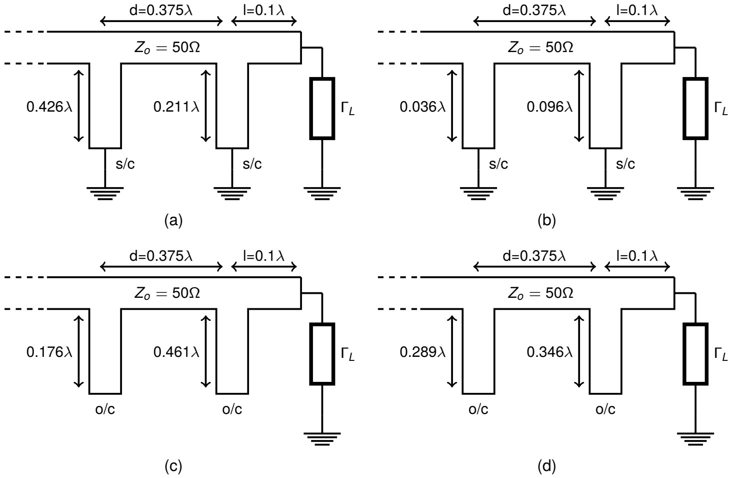


Figure 20 : Four possible double stub matching networks for the load $\Gamma_L = 0.667\angle 90^o$ implemented in microstrip : (a) Solution 1 (points D,E) s/c stubs, (b) Solution 2 (points F,G) s/c stubs, (c) Solution 1 (points D,E) o/c stubs, (d) Solution 2 (points F,G) o/c stubs

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Triple Stub Matching

- Double stub matching is not possible for certain values of load impedance and stub placements. One solutions is to add a third stub.
- Triple stub matching is rarely used as a design technique in fixed media, such as microstrip, but commercial "Triple Stub Tuners" are available as standard components in waveguide or co-axial media.
- These tuners are frequently implemented with three stubs spaced at unequal intervals, which ensures that any value of passive load can be matched. Short circuit stubs are mainly used in co-axial or waveguide, for ease of adjustment.
- Examples of waveguide and co-axial triple stub tuners are shown in figure 21.

(a) Waveguide triple stub tuner

(b) co-axial triple stub tuner

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Quarter-Wavelength Transformer (QWT)

Consider a situation where we have connected a length of transmission line of length $\lambda/4$ and characteristic impedance Z_1 in front of the load :

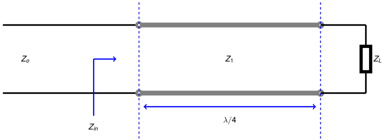


Figure 22 : Quarter Wave Transformer

From equation (**??**), the input impedance of any lossless line section, of characteristic impedance Z_1, is given by :

$$Z_{in} = Z_1 \left(\frac{Z_L + jZ_1 \tan(\beta l)}{Z_1 + jZ_L \tan(\beta l)} \right) \tag{33}$$

But in figure 22 we have set $l = \lambda/4$, so (33) now becomes:

$$Z_{in} = Z_1 \cdot \frac{Z_L + jZ_1 \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right)}{Z_1 + jZ_L \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right)} = Z_1 \cdot \frac{Z_L + jZ_1 \tan\left(\frac{\pi}{2}\right)}{Z_1 + jZ_L \tan\left(\frac{\pi}{2}\right)} \tag{34}$$

Since $\tan(\pi/2) = \infty$ (34) reduces to:

$$Z_{in} = \frac{Z_1^2}{Z_L} \tag{35}$$

Quarter-Wavelength Transformer (QWT)

Equation (35) tells us that a quarter wave line section can act as a matching network between a load, Z_L, and a required input impedance, Z_{in}, provided that the line section has a characteristic impedance given by :

$$\boxed{Z_1 = \sqrt{Z_L Z_{in}}} \tag{36}$$

Such a line section is called a *Quarter Wave Transformer* (QWT).

Equation (43) implies that a real QWT can only be used to match Resistive loads, but we will show that the QWT can also be used to match complex loads with a little modification.

Using a QWT match a complex load

- Equation (43) implies that, since the characteristic impedance of the QWT must be real, this QWT can only be used to match purely resistive loads, ($Z_L = R_L + j0$). To match complex loads, we would need a transmission line with a complex characteristic impedance, i.e. the line would have to be lossy. This is generally undesirable.
- One alternative is to transform the complex load into a real quantity by inserting a transmission line section between the load and the QWT, as illustrated, in microstrip form, in figure 23.
- For convenience we will assume that the added line section has the system characteristic impedance, Z_o.

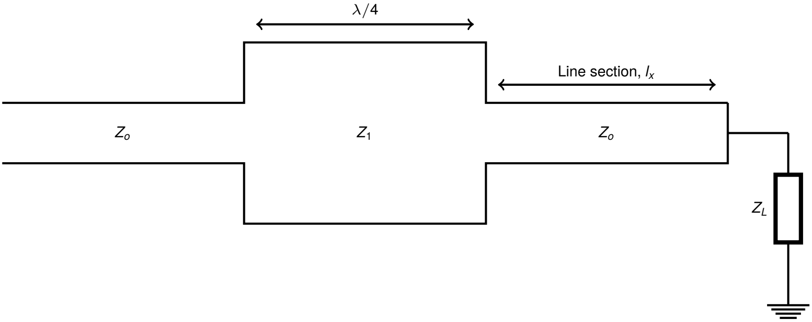


Figure 23 : Quarter Wave Transformer (QWT) with a complex load

Using a QWT match a complex load

The effect of adding the extra line section, l_x, in figure 23 can be demonstrated by considering the input impedance, Z_{in} of any line section terminated by a load Z_L. Z_{in} is given by equation (**??**), which can be expressed in normalised terms as :

$$z_{in} = \frac{Z_L + jt}{1 + jZ_L t} \tag{37}$$

where, again:

$$t = \tan\left(2\pi \frac{l_x}{\lambda}\right)$$

Since we need the input impedance of the added line section in figure 23 to be purely real, we set $\Im(z_{in}) = 0$ in (37), which results in :

$$\frac{(x_L + t)(1 - x_L t) - r_L^2 t}{(1 - x_L)^2 + r_L^2 t} = 0 \tag{38}$$

where $Z_L = r_L + jx_L$.

Using a QWT match a complex load

- Figure 24 illustrates how such a matching network can be designed using a Smith chart.
- We plot the normalised load impedance, z_L, on the chart at point 'A'. We then draw the constant VSWR circle through the point A, and we note that this circle crosses the real axis of the chart at two points, B and C. At both these points the input impedance of the line section is purely real.
- We note that at point B $Z_p > Z_o$ and at point C $Z_p < Z_o$, the lengths of series line required to achieve this are given by the distances AB and AC=AB+0.25λ in figure 24.

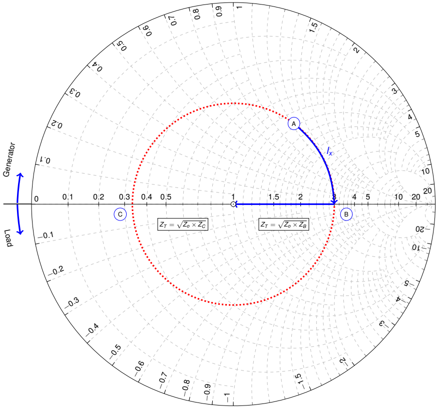


Figure 24 : QWT with complex load : graphical solution

Using a QWT match a complex load

- Once the length of series line l_x has been chosen, the QWT is then used to transform Z_B or Z_C into Z_o. Choosing point B will require a transformer with a characteristic impedance greater than Z_o and choosing point C will result in a transformer with Z_T less than Z_o.
- Whichever point is chosen, Z_T is computed using (**??**).
- If the QWT is being implemented in microstrip medium, the lower value of Z_T is generally preferred because low impedance lines (i.e. thicker traces) are easier to reproduce accurately.

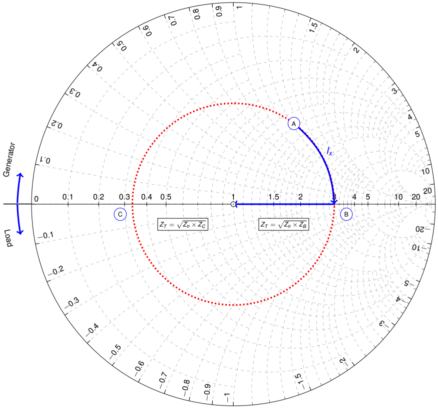


Figure 25 : QWT with complex load : graphical solution

Quarter-Wavelength Transformer (QWT)

Equation (38) implies:

$$(x_L + t)(1 - x_L t) - r_L^2 t = 0 \tag{39}$$

Rearranging (39), and noting that $|Z_L|^2 = r_L^2 + x_L^2$, results in the following quadratic in t:

$$x_L t^2 - t(1 - |Z_L|^2) - x_L = 0 \tag{40}$$

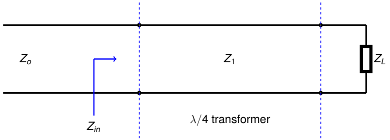
We can now determine t as follows :

$$t = \frac{(1 - |Z_L|^2) \pm \sqrt{(1 - |Z_L|^2)^2 + 4x_L^2}}{2x_L} \tag{41}$$

The two solutions embodied in (41) represent the two intersections of the constant VSWR circle through Z_L with the real axis. One of these intersections gives $Re(z_{in}) < Z_o$ and the other gives $Re(z_{in}) > Z_o$. With a given value of t, we determine the electrical length of the line section, l_x, from :

$$\frac{l_x}{\lambda} = \frac{1}{2\pi} \tan^{-1}(t)$$

Quarter-Wavelength Transformer (QWT)



From the previous slide we have :

$$Z_{in} = \frac{Z_1^2}{Z_L} \tag{42}$$

We can therefore us a QWT as a matching network between a load, Z_L, and a required input impedance, Z_{in}, provided that the QWT has a characteristic impedance given by :

$$\boxed{Z_1 = \sqrt{Z_L Z_{in}}} \tag{43}$$

This implies that a real QWT can only be used to match Resistive loads, Z_L.

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Bandwidth of Single Stub Matching Networks

The fractional bandwidth for the previous single stub matching example is shown for open circuit stubs in figure 26.

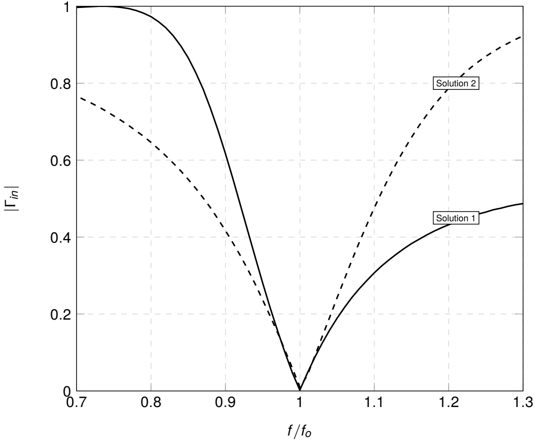


Figure 26 : Single stub matching network bandwidth: open circuit stubs

Bandwidth of Single Stub Matching Networks

The fractional bandwidth for the previous single stub matching example is shown for short circuit stubs in figure 27.

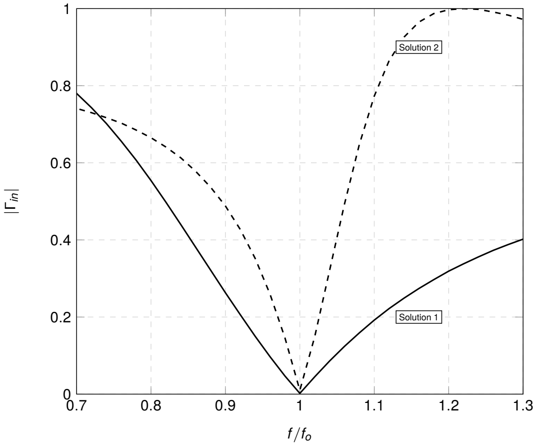


Figure 27 : Single stub matching network bandwidth: short circuit stubs

Bandwidth of Single Stub Matching Networks

In general, the single stub matching solution with the shortest combination of line and stub lengths will provide the widest bandwidth [1]. This can be demonstrated, with reference to example **??** in section **??**, by considering an arbitrary value of $|\Gamma_{in}|$, say 0.2. At this value of reflection coefficient, the various fractional bandwidths of the single stub matching network are shown in table 3.

Table 3 : Fractional bandwidth of a typical single stub matching network

	Open circuit stub	Short circuit stub
Solution 1	9%	19%
Solution 2	8%	6%

Bandwidth of Double Stub Matching Networks

Figure 28 shows fractional bandwidth for a typical open circuit double stub matching network.

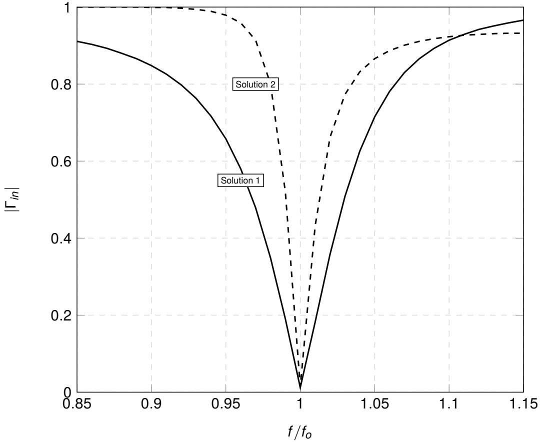


Figure 28 : Double stub matching network bandwidth: open circuit stubs

Bandwidth of Double Stub Matching Networks

Figure 29 shows fractional bandwidth for a typical short circuit double stub matching network.

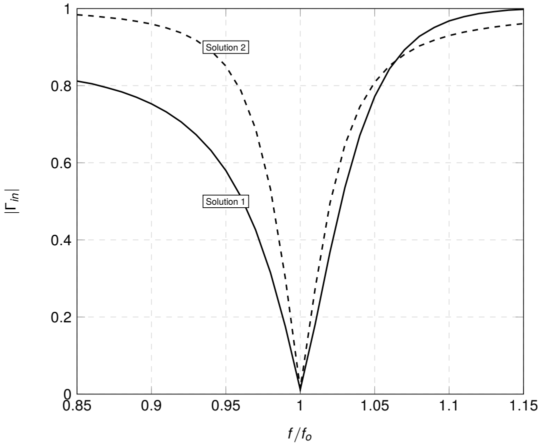


Figure 29 : Double stub matching network bandwidth : short circuit stubs

Bandwidth of Double Stub Matching Networks

We can compare the fractional bandwidths of the double stub solutions of figure 28 and figure 29 by adopting a similar approach to that taken with the single stub solutions, only this time, because the bandwidth is narrower, we will choose a higher value of $|\Gamma_{in}| = 0.3$. The results are shown in table 4.

Table 4 : Fractional bandwidth of a typical double stub matching network

	Open circuit stubs	Short circuit stubs
Solution 1	3%	4%
Solution 2	1%	2%

QWT Bandwidth

A closed form expression for the fractional bandwidth of a single section transformer has been derived[2] as follows :

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{(1-\Gamma_m^2)}}\frac{2\sqrt{Z_L Z_o}}{|Z_L - Z_o|}\right] \quad (44)$$

Where Γ_m is the maximum reflection coefficient that we can accept as defining a 'good' match.

We can see from (44) that the fractional bandwidth of the QWT increases as Z_L becomes closer to Z_o, as outlined above. For a value of $|\Gamma_{in}| = 0.1$, the fractional bandwidth of the QWT shown in figure 30 with various loads is summarised in table 5.

Table 5 : Fractional bandwidth of a typical quarter wave transformer

Load	Fractional Bandwidth
$Z_L = 150\Omega$ or 16.67Ω	22%
$Z_L = 100\Omega$ or 25Ω	36%
$Z_L = 75\Omega$ or 37.5Ω	66%

QWT Bandwidth

- The Quarter Wave Transformer is an inherently narrow band matching network, since (by definition) it is only exactly a quarter wavelength at one frequency.
- It turns out, however, that the closer R_L is to characteristic impedance Z_o, the wider the bandwidth of the quarter wavelength transformer.

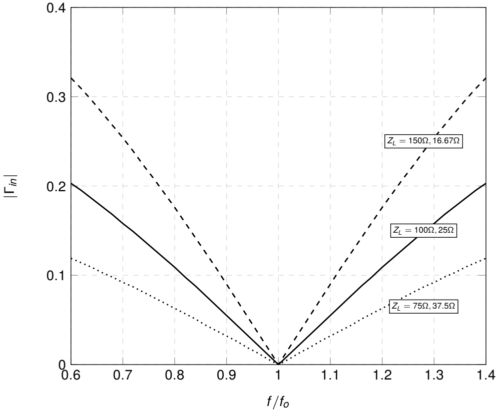


Figure 30 : Quarter wave transformer bandwidth

Quarter Wave Transformer design example

Design a Quarter wave transformer to match a load of 10Ω to a 50Ω transmission line. Determine the fractional bandwidth of this matching network if we define the VSWR for a good match to be less than 1.6.

Solution : We can match this real impedance using a QWT with a characteristic impedance of :

$$Z_T = \sqrt{10 \times 50} = 22.36\Omega$$

A VSWR of 1.6 corresponds to a reflection coefficient magnitude of :

$$\Gamma_m = \frac{VSWR - 1}{VSWR + 1} = \frac{1.6 - 1}{1.6 + 1} = 0.231$$

The fractional bandwidth can now be computed from (44) as :

$$\begin{aligned}
\frac{\Delta f}{f_o} &= 2 - \frac{4}{\pi} cos^{-1}\left[\frac{0.231}{\sqrt{(1 - 0.231^2)}} \frac{2\sqrt{10 \times 50}}{|10 - 50|}\right] \\
\frac{\Delta f}{f_o} &= 0.169
\end{aligned}$$

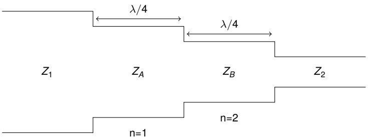
The fractional bandwidth of this transformer is therefore 16.9%.

Multi-Stage Quarter Wave Transformers

- There is a natural bandwidth limitation of quarter-wave transformers, because you only get exactly a quarter wavelength at one frequency
- Lower frequencies see less than a quarter-wave, higher frequencies see more.
- In order to achieve matching over a broader bandwidth, we add multiple quarter-wave transformers in a series, so that the impedance mismatch that each transformer is correcting becomes less and less.
- The bandwidth limitation of the single quarter-wave section can be overcome by using multiple sections in series, with impedances chosen to provide the desired response (e.g. maximally flat, Tchebyscheff etc.)
- The improvement in bandwidth carries a cost of increased physical size, which may be a problem in some circumstances.

Example Multi-Stage Transformers

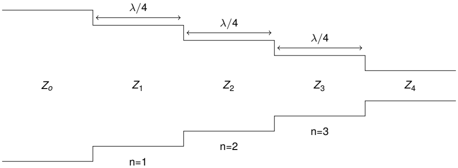
'2-quarter-wave' transformer :



$$Z_A = Z_1 \left[\frac{Z_2}{Z_1}\right]^{\frac{1}{4}} \tag{45}$$

$$Z_A = Z_1 \left[\frac{Z_2}{Z_1}\right]^{\frac{3}{4}} \tag{46}$$

'3-quarter-wave' transformer :



$$Z_A = Z_1 \left[\frac{Z_2}{Z_1}\right]^{\frac{1}{4}} \tag{47}$$

$$Z_A = Z_1 \left[\frac{Z_2}{Z_1}\right]^{\frac{3}{4}} \tag{48}$$

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