# Lecture 14 - Low Noise Amplifier Design

***Microwave Active Circuit Analysis and Design***

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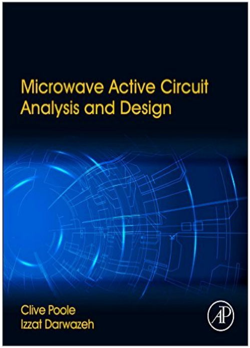

# Intended Learning Outcomes

- Knowledge
  - Understand the most important sources of electrical noise, such as thermal noise, shot noise and flicker noise, and their characteristics.
  - Know the definition of noise factor and noise figure for a two-port network.
  - Understand the relationship between noise factor and effective noise temperature.
  - Understand the relationship between noise figure and source termination for a single stage and multi-stage amplifiers.
  - Understand the basic principles of noise figure measurement and transistor noise characterisation.
- Skills
  - Be able to design a single stage microwave transistor amplifier having the minimum possible noise figure.
  - Be able to design a single stage microwave transistor amplifier having a specified noise figure and gain.
  - Be able to calculate the overall noise figure of a receiver chain.
  - Be able to design a single stage microwave transistor amplifier having the minimum noise measure, and thereby design a multi-stage amplifier having the minimum possible noise figure.
  - Be able to design a single stage microwave transistor amplifier with a specified noise measure and gain, and thereby design a multi-stage amplifier having a specified noise figure.

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# Thermal Noise

- The most common form of intrinsic electrical noise in circuits is *thermal noise*, which is generated by the random thermal motion of electrons within any conducting or semi-conducting material.
- This thermal motion would cease to exist if the material is 'properly frozen', i.e. taken down to absolute zero (0 kelvin). Thermal noise is also known as *Johnson noise* after J.B. Johnson who first observed the phenomenon in 1927 [9].

The mean square value of thermal noise voltage and current in a resistor, $R$ (in $\Omega$), in a bandwidth $\Delta f$ (in Hz) and at an absolute operating temperature $T_o$ (in kelvin) are given, respectively, by the two equations below :

$$\overline{|v_{nt}|^2} = 4k_B T_o R \Delta f (in\ units\ of\ V^2) \tag{1}$$

$$\overline{|i_{nt}|^2} = \frac{4k_B T_o \Delta f}{R} (in\ units\ of\ A^2) \tag{2}$$

where $k_B$ is Boltzmann's constant ($= 1.3806488 \times 10^{-23}$ joules per kelvin).

# Thermal Noise

- $\overline{|v_{nt}|^2}$ and $\overline{|i_{nt}|^2}$ are equal to the variances of the Gaussian distributions that describe the noise voltage and current, respectively.
- It is convenient for circuit designers to express noise in units of volts or amperes. These are expressed as Root Mean Square (RMS) values.

The RMS voltage, $v_{n_t}$, and the corresponding current, $i_{n_t}$, due to thermal noise in a resistance $R$ (in $\Omega$) may simply be obtained by taking the square roots of the quantities in (1) and (2), giving :

$$v_{n_t} = \sqrt{4k_B T_o R \Delta f} \tag{3}$$

$$i_{n_t} = \sqrt{\frac{4k_B T_o \Delta f}{R}} \tag{4}$$

To find the thermal noise power generated by an arbitrary resistor $R$, we can apply one or both of (3) and (4). We then have the noise power, $P_{n_t}$, generated by the resistor $R$ as :

$$P_{n_t} = v_{n_t} \cdot i_{n_t} \tag{5}$$

$$= \frac{v_{n_t}^2}{R} = i_{n_t}^2 R \tag{6}$$

$$= 4k_B T_o \Delta f \tag{7}$$

# Thermal Noise

- One way of understanding equation (5) is to think of $P_{n_t}$ as the power dissipated in the noise generating resistor itself when it is terminated by a short circuit.
- This power is directly proportional to the bandwidth and the absolute temperature, but is independent of the resistor value.
- What we are primarily interested in is the amount of noise power that will be transferred to an external circuit.
- According to the maximum power transfer theorem, the maximum noise power will be extracted from the resistor, $R$, when the equivalent resistance of the external circuit is also equal to $R$, as illustrated in figure 1.

The noise voltage across the external load resistor in figure 1, is $v_n/2$, where $v_n$ is defined by (3). The maximum available noise power from $R$ is therefore given by:

$$P_{n_t(max)} = k_B T_o \Delta f \qquad (8)$$

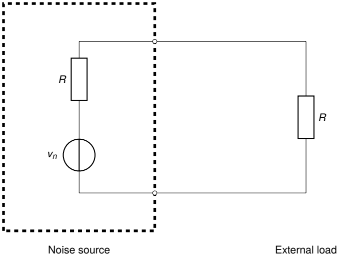


Figure 1 : Maximum noise power extraction from a resistor $R$.

# Thermal Noise

- The load resistor in figure 1 is also a source of thermal noise, and that each one of the two participating resistors generates and dissipates noise in both itself and in the other resistor.
- This does not alter the validity of (8), since, as the two resistors are physically separate entities, their noise voltages are not correlated and so do not add constructively.

We can express the noise power in dBm as follows:

$$P_{n_t}(dBm) = 10\log_{10}(k_B T_o \Delta f \times 1,000) \tag{9}$$

where the factor of 1,000 in (9) is present because dBm is a ratio of the power to 1mW. We can separate out the bandwidth element of (9) from the constant elements as follows:

$$P_{n_t}(dBm) = 10\log_{10}(k_B T_o \times 1,000) + 10\log_{10}(\Delta f) \tag{10}$$

If we take $T_o$ to be room temperature (290 K), (10) can be written in a compact form as:

$$P_{n_t}(dBm) \approx -174 + 10\log_{10}(\Delta f) \tag{11}$$

If we take the bandwidth to be 1 Hz, (11) gives us the **Thermal Noise Floor** as -174 dBm at room temperature.

# Shot noise

- Shot noise in electronic devices arises from the discrete nature of electric current and relates to the arrival of charge carriers at a particular place, i.e. when electrons cross some type of physical 'gap', such as a *pn* or Schottky junction.
- Unlike thermal noise, shot noise is characterised by the Poisson distribution[3], which describes the occurrence of independent and discrete random events.
- When the number of events is sufficiently high, as in the case of the flow of electrons in a circuit with 'normal' operating currents, the Poisson distribution resembles the Gaussian distribution.
- For most practical cases, therefore, we usually assume that the shot noise and thermal noise have the same distribution.
- This makes our circuit analysis and design more straightforward. In other words, we simply add the shot noise component to the thermal noise component.
- Shot noise, just like thermal noise, can be characterised as 'white noise' due to its flat power spectral density.

# Shot noise

As the shot noise has its physical origin in electrons crossing a junction, it is normally expressed in terms of electron flow, in other words, current. The RMS value of the shot noise current is given by[5] :

$$i_{n_s} = \sqrt{2Iq\Delta f} \tag{12}$$

where $I$ is the DC current, $q$ is the electron charge, and $\Delta f$ is the bandwidth in Hz.

In all active circuits where semiconductor devices are biased, shot noise exists and has to be accounted for by designers.

We note from (12) that shot noise is not a function of temperature, unlike thermal noise. We should also note that conductors and resistors do not exhibit shot noise because there is no ’gap’ as such.

# Flicker noise

- In addition to thermal noise semiconductor devices also exhibit a particular type of noise called **flicker noise** or $1/f$ noise, after its frequency characteristic which falls off steadily as frequency increases from zero.
- Because of its spectral characteristics flicker noise is sometimes referred to as 'Pink' noise (as opposed to thermal and shot noise which have a 'white' spectrum).
- Unlike other types of noise, $1/f$ noise is a non stationary random process[10], in other words its statistics vary with time.
- The **flicker noise corner frequency**, $f_c$, defines the boundary between flicker noise dominant and thermal noise dominant regions in the frequency domain. In fact, $1/f$ noise has spectral characteristics that can be described as comprising a number of $1/f^{\alpha}$ curves with various cut-off frequencies depending upon the value of the integer $\alpha$.
- The corner frequencies and the actual spectral density of $1/f$ noise depend on the type of material used to construct a semiconductor device, the device geometry and the bias.
- Generally, both the $1/f$ noise spectral density and the corner frequency increase with bias current. The corner frequencies range from tens of Hz to tens of kHz[1].

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# Noise Factor

The noise factor of a two-port network is calculated as a simple ratio of input SNR to output SNR, as follows:

$$F = \frac{SNR_{in}}{SNR_{out}} \tag{13}$$

For any real world device or circuit in which internal noise will be generated as described in the previous sections, the input SNR will never be less than the output SNR. The noise factor, $F$, for such a device can therefore never be less than 1.

The noise factor is most often presented in the form of the *Noise Figure*, which is simply the noise factor expressed in dB as follows:

$$F_{dB} = 10\log_{10}(F) = 10\log_{10}\left(\frac{SNR_{in}}{SNR_{out}}\right) \tag{14}$$

# Noise Temperature

- Since any changes in temperature will affect the noise power, the formulae in the previous slides are valid at a specified operating temperature, $T_o$.
- We can therefore define something called the *effective noise temperature* of any device or circuit as being the absolute temperature at which a perfect resistor, of equal resistance to the device or circuit, would generate the same noise power as that device or circuit at room temperature.
- We can also define the effective input noise temperature of an amplifier or other two-port network as the source noise temperature that would result in the same output noise power, when connected to an ideal 'noise-free' network or amplifier, as that of the actual network or amplifier connected to a noise-free source.

The relationship between noise factor and noise temperature $T_e$ of a device is as follows:

$$F = 1 + \frac{T_e}{T_o} \tag{15}$$

Where $T_o$ is the actual operating temperature (in kelvin).

# Noise figure vs Noise temperature

The relationship between noise figure in dB and noise temperature is defined by:

$$F = \left(1 + \frac{T_e}{T_o}\right) \tag{16}$$

Or in dB terms:

$$F_{dB} = 10\ \log_{10}\left(1 + \frac{T_e}{T_o}\right) \tag{17}$$

One reason for using noise temperature as a figure of merit is that it provides greater resolution at very small values of noise factor (where $F \approx 1$). For this reason extremely low noise amplifiers may be characterised by their effective noise temperature.

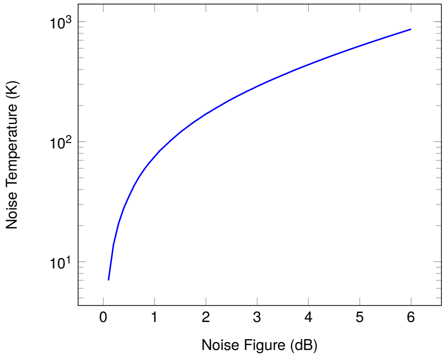


Figure 2 : Noise Temperature versus Noise Figure (dB)

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# Representation of noise in active two-port networks

A noisy two-port can be represented by a noise-free two-port with external noise sources at the input and output, as shown below:

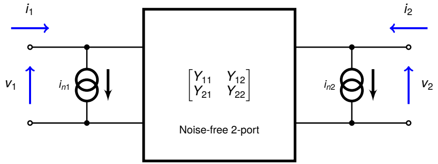


Figure 3 : Y-parameter noise representation of noisy two-port

This noisy two-port may be described in terms of its $Y$-parameters as follows :

$$i_1 = Y_{11}v_1 + Y_{12}v_2 + i_{n1}$$
$$i_2 = Y_{21}v_1 + Y_{22}v_2 + i_{n2}$$

The analysis is simplified if we represent the noisy two-port in terms of its ABCD matrix, so that both noise sources may now be located at the input port, as shown below.

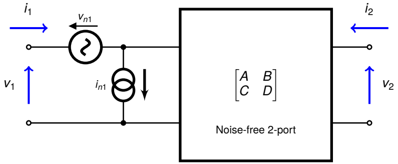


Figure 4 : ABCD noise representation of noisy two-port

The ABCD representation can be described by the following set of equations:

$$\begin{aligned} i_1 &= AV_2 + BI_2 + i_{n1} \\ v_1 &= CV_2 + DI_2 + v_{n1} \end{aligned} \tag{18}$$

# Representation of noise in active two-port networks

The noise-free two-port of figure **??** has the same signal-to-noise ratio at its input and output. Therefore, the noise figure of the overall two port can be derived by considering the input noise network alone ($v_{n1}$ and $i_{n1}$).

Consider the input noise network in figure **??** connected to a source of internal admittance $Y_S = G_S + jB_S$ and a noise current $i_{ns}$ which is uncorrelated with $v_{n1}$ or $i_{n1}$. Given (**??**), we can replace the noise voltage source, $v_{en1}$, with an equivalent current source $Y_{cor}v_{n1}$, as shown in figure 5.

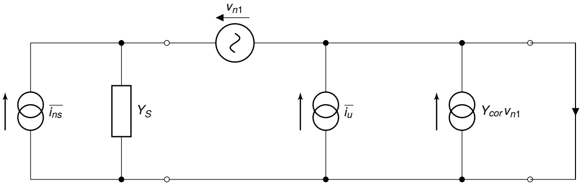


Figure 5 : Input noise equivalent model

The mean square short circuit output current of this network is given by :

$$\overline{|i_{ntot}|^2} = \overline{|i_{ns} + i_u + v_{n1}(Y_S + Y_{cor})|^2} \tag{19}$$

# Representation of noise in active two-port networks

Since the components of the right hand side of equation (19) are uncorrelated with each other, the mean-square value of $i_{ntot}$ is equal to the sum of the mean-square values of the components. We can therefore rewrite equation (19) as:

$$\overline{|i_{ntot}|^2} = \overline{|i_{ns}|^2} + \overline{|i_u|^2} + \overline{|e_{n1}|^2}|Y_S + Y_{cor}|^2 \quad (20)$$

We defined the noise factor of a two-port network in (13). Another definition of the Noise Factor is the ratio of the total output noise power per unit bandwidth to the total input noise power per unit bandwidth[8]. Using this definition, the noise factor of the circuit of figure **??** may be written as :

$$F = \frac{\overline{|i_{ntot}|^2}}{\overline{|i_{ns}|^2}} \quad (21)$$

Applying equation (20) this becomes:

$$F = 1 + \frac{\overline{|i_u|^2}}{\overline{|i_{ns}|^2}} + \frac{\overline{|e_{n1}|^2}}{\overline{|i_{ns}|^2}}|Y_S + Y_{cor}|^2 \quad (22)$$

# Representation of noise in active two-port networks

The various voltage and current components of equation (22) may all be defined in terms of equivalent noise resistances and conductances as follows :

$$\overline{|i_{ns}|^2} = 4k_B T_o G_S \Delta f \tag{23}$$

$$\overline{|i_u|^2} = 4k_B T_o G_u \Delta f \tag{24}$$

$$\overline{|e_{n1}|^2} = 4k_B T_o R_n \Delta f \tag{25}$$

Substituting these definitions (23) to (25) into (22) results in:

$$F = 1 + \frac{G_u}{G_S} + \frac{R_n}{G_S}|Y_S + Y_{cor}|^2 \tag{26}$$

or

$$F = 1 + \frac{G_u}{G_S} + \frac{R_n}{G_S}[(G_S + G_{cor})^2 + (B_S + B_{cor})^2] \tag{27}$$

# Representation of noise in active two-port networks

Substituting these definitions (23) to (25) into (22) results in:

$$F = 1 + \frac{G_u}{G_S} + \frac{R_n}{G_S}|Y_S + Y_{cor}|^2 \qquad (28)$$

or

$$F = 1 + \frac{G_u}{G_S} + \frac{R_n}{G_S}[(G_S + G_{cor})^2 + (B_S + B_{cor})^2] \qquad (29)$$

The noise factor of the two-port is therefore an explicit function of the source admittance and depends upon four parameters, $G_u, R_n, G_{cor}$ and $B_{cor}$.

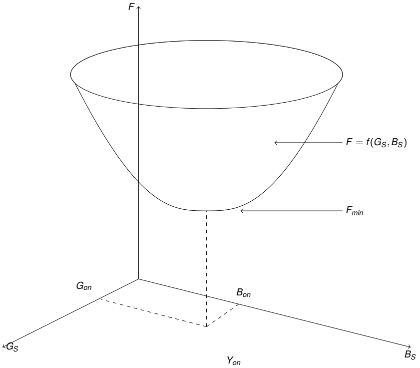


Figure 6 : Noise factor, $F$, as a function of $Y_S$

# Representation of noise in active two-port networks

By employing equation (**??**) this becomes:

$$F = F_{min} + \frac{R_n}{G_S}[(G_S - G_{on})^2 + (B_S - B_{on})^2] \tag{30}$$

This equation is more often written in its equivalent form:

$$\boxed{F = F_{min} + \frac{R_n}{G_S}|Y_S + Y_{on}|^2} \tag{31}$$

Where $Y_{on}$ is the optimum source termination ($Y_{on} = G_{on} + jB_{on}$).

# Representation of noise in active two-port networks

In the microwave frequency range we are more accustomed to working with reflection coefficients than with impedances or admittances. Equation (31) may be translated into the source reflection coefficient plane by using the relationships:.

$$Y_S = \frac{1}{Z_o}\frac{(1-\Gamma_S)}{(1+\Gamma_S)}$$
$$Y_{on} = \frac{1}{Z_o}\frac{(1-\Gamma_{on})}{(1+\Gamma_{on})} \tag{32}$$

This leads to the equation:

$$\boxed{F = F_{min} + 4r_n \frac{|\Gamma_S - \Gamma_{on}|^2}{|1+\Gamma_{on}|^2(1-|\Gamma_S|^2)}} \tag{33}$$

Where $r_n$ is the normalised equivalent input noise resistance, which is defined as :

$$r_n = \frac{R_n}{Z_o} \tag{34}$$

The four scalar parameters, $F_{min}$, $|\Gamma_{on}|$, $\angle\Gamma_{on}$ and $R_n$ are known as the *Noise Parameters* and are often specified in manufacturer's data sheets for a given microwave transistor, alongside the S-parameters.

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# Single-stage low noise amplifier design

We can build on the two-port noise analysis of the previous slides to set out a design methodology for low noise microwave transistor amplifiers.

We rely on the noise parameters that are usually provided by the device manufacturer, but can be measured if necessary.

There are two real and one complex parameter we need for this purpose, being the parameters used in equation (33), namely :

- The minimum noise figure : $F_{min}$ in dB.
- The equivalent noise resistance : $R_n$ in $\Omega$.
- The optimum source termination : $\Gamma_{on}$ (which is dimensionless).

As a reminder, we will use the symbol $R_n$ to represent the ohmic value of equivalent noise resistance and the symbol $r_n$ to represent the normalised value.

# Circles of constant noise figure

- A graphical representation of the effect of variations in $\Gamma_s$ on the noise factor of an amplifier provides a means of assessing the "trade-off" between noise figure and gain, when plotted on the same axes.
- It can be shown that loci of constant noise factor obtained from equation (33) are circles in the source reflection coefficient plane[7].
- We will focus instead on the reflection coefficient based approach, which is more widely used today.

We start by considering equation (33):

$$F = F_{min} + 4r_n \frac{|\Gamma_S - \Gamma_{on}|^2}{|1 + \Gamma_{on}|^2(1 - |\Gamma_S|^2)} \tag{33}$$

Rearranging equation (33) above gives :

$$\frac{(F - F_{min})|1 + \Gamma_{on}|^2}{4r_n} = \frac{|\Gamma_S - \Gamma_{on}|^2}{(1 - |\Gamma_S|^2)} \tag{35}$$

which can be rearranged as :

$$N_i(1 - |\Gamma_S|^2) = |\Gamma_S|^2 + |\Gamma_{on}|^2 - \Gamma_S^*\Gamma_{on} - \Gamma_S\Gamma_{on}^* \tag{36}$$

Where :

$$N_i = \frac{(F - F_{min})|1 + \Gamma_{on}|^2}{4r_n} \tag{37}$$

# Circles of constant noise figure

Rearranging equation (36) leads to :

$$|\Gamma_S|^2 - \Gamma_S \frac{\Gamma_{on}^*}{(1+N_i)} - \Gamma_S^* \frac{\Gamma_{on}}{(1+N_i)} = \frac{N_i - |\Gamma_{on}|^2}{(1+N_i)} \tag{38}$$

Equation (**??**) can be rearranged into the form of a circle in the $\Gamma_s$ plane, that is to say, it is of the form :

$$|\Gamma_S - C_{Sn}| = \gamma_{Sn}^2 \tag{39}$$

Where the center is given by:

$$\boxed{C_{Sn} = \frac{\Gamma_{on}}{1+N_i}} \tag{40}$$

and the radius is given by:

$$\boxed{\gamma_{Sn} = \frac{\sqrt{N_i^2 + N_i(1 - |\Gamma_{on}|^2)}}{1+N_i}} \tag{41}$$

# Design Example 1 : Avago ATF-34143 HEMT

Problem : Draw constant noise figure circles for $F = 1.4dB$, for $F = 2dB$ and $F = 3dB$ on the source plane for the Avago ATF-34143 Low Noise HEMT operating at 10GHz, and hence, or otherwise, determine the lowest possible noise figure commensurate with the maximum gain available from this device. The S-parameters and noise parameters of the device with bias conditions $V_{DS} = 3V$, $I_{DS} = 40mA$ are as follows :
S-parameters :

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0.760\angle 28 & 0.144\angle -84 \\ 1.647\angle -84 & 0.410\angle 23 \end{bmatrix}$$

Noise parameters :

$$F_{min} = 1.22dB$$
$$\Gamma_{on} = 0.61\angle -39^o$$
$$R_n = 25\Omega$$

# Design Example 1 : Avago ATF-34143 HEMT

Solution : Firstly, we need to investigate the stability of the device, for which we will use the Edwards Sinsky stability criteria defined by (**??**) and (**??**), i.e.:

$$\mu_1 = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} = \frac{0.4224}{0.3134} = 1.1887$$

$$\mu_2 = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta S_{22}^*| + |S_{12}S_{21}|} = \frac{0.8319}{0.8246} = 1.0435$$

Since both $\mu_1$ and $\mu_2$ are greater than 1 we conclude that the device is unconditionally stable, so we are free to choose any terminating impedances lying within the $|\Gamma| = 1$ boundary of the source and load plane Smith Charts. Maximum available gain occurs when the source and load are simultaneously conjugately matched.

# Design Example 1 : Avago ATF-34143 HEMT

The necessary terminating reflection coefficients are determined as follows :

$$\Gamma_{ms} = C_1^* \left[ \frac{B_1 + \sqrt{B_1^2 - 4|C_1|^2}}{2|C_1|^2} \right] = 0.5601\angle -34^o \left[ \frac{1.1413 + \sqrt{1.1413^2 - 4 \times 0.5601^2}}{2 \times 0.5601^2} \right]$$

$$= 0.8233\angle -34^o$$

$$\Gamma_{ml} = C_2^* \left[ \frac{B_2 + \sqrt{B_2^2 - 4|C_2|^2}}{2|C_2|^2} \right] = 0.1182\angle -97^o \left[ \frac{0.3223 + \sqrt{0.3223^2 - 4 \times 0.1182^2}}{2 \times 0.1182^2} \right]$$

$$= 0.4364\angle -97^o$$

Where :

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 = 1.1413$$
$$C_1 = S_{11} - \Delta S_{22}^* = 0.5601\angle 34^o$$
$$B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\Delta|^2 = 0.3223$$
$$C_2 = S_{22} - \Delta S_{22}^* = 0.1182\angle 97^o$$

# Design Example 1 : Avago ATF-34143 HEMT

With the above conjugate terminations the Maximum Available Gain (MAG) of the device, from (**??**) is:

$$MAG = \frac{|S_{21}|}{|S_{12}|}\left[K - \sqrt{K^2 - 1}\right] = \frac{1.647}{0.144}\left[1.1016 - \sqrt{1.1016^2 - 1}\right]$$

$$= 7.3151 = 8.6dB$$

Where $K$ is the Rollett stability factor.

# Design Example 1 : Avago ATF-34143

In order to draw the constant noise figure circles for $F = 1.4dB$, $F = 2dB$ and $F = 3dB$ on the $\Gamma_S$ plane, the first step is to calculate the parameter $N_i$, as defined by (37) for the various values of noise figure. For example, for $F = 1.4dB$ we have :

$$N_i = \frac{(F - F_{min})|1 + \Gamma_{on}|^2}{4r_n}$$

$$= \frac{(10^{(1.4/10)} - 10^{(1.22/10)})|1 + 0.61\angle - 39|^2}{4 \times 25/50}$$

$$= \frac{0.0560 \times 2.3202}{2} = 0.0650$$

Similarly, we calculate $N_i$ for $F = 2dB$ and $F = 3dB$ to be 0.3023 and 0.7783 respectively.

# Design Example 1 : Avago ATF-34143 HEMT

Employing (40) and (41) we can now calculate the centres and radii of the three noise figure circles as follows:

1.4dB noise figure circle :

$$C_{Sn_{1.4}} = \frac{0.61\angle -39}{1+0.0650} = 0.5730\angle -39$$

$$\gamma_{Sn_{1.4}} = \frac{\sqrt{(0.0042 + 0.0650 \times 0.6279)}}{1+0.0650} = \boxed{0.1993}$$

2dB noise figure circle :

$$C_{Sn_{2.0}} = \frac{0.61\angle -39}{1+0.3023} = 0.4684\angle -39$$

$$\gamma_{Sn_{2.0}} = \frac{\sqrt{(0.0914 + 0.3023 \times 0.6279)}}{1+0.3023} = \boxed{0.4072}$$

3dB noise figure circle :

$$C_{Sn_{3.0}} = \frac{0.61\angle -39}{1+0.7783} = 0.3430\angle -39$$

$$\gamma_{Sn_{3.0}} = \frac{\sqrt{(0.6058 + 0.7783 \times 0.6279)}}{1+0.7783} = \boxed{0.5883}$$

# Design Example 1 : Avago ATF-34143 HEMT

- The noise figure circles are now plotted on the Smith chart in figure 7, together with the optimum source termination , $\Gamma_{on}$, which is basically the centre of the noise figure circle of zero radius (i.e. when we set $F = F_{min}$ in (41) we get $\gamma_{Sn} = 0$).
- We have also plotted the optimum source termination for maximum available gain, $\Gamma_{ms}$, on figure 7 and we can see that this lies between the $F = 1.4dB$ and $F = 2dB$ noise figure circles, indicating that the noise figure of the device, when simultaneously conjugately matched for maximum gain, will have a noise figure between $1.4dB$ and $2dB$.

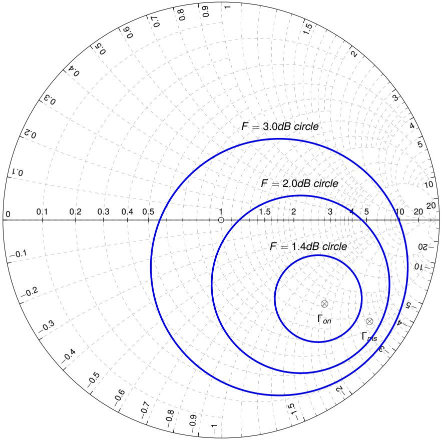



Figure 7 : Constant noise figure circles for Avago ATF-34143 at 10GHz ($V_{DS} = 3V$, $I_{DS} = 40mA$)

# Design Example 1 : Avago ATF-34143 HEMT

We can calculate the exact noise figure of the simultaneously conjugately matched device by employing equation (33). If the input port is matched with $\Gamma_{ms}$, the noise figure will be:

$$
\begin{aligned}
F =& F_{min} + 4r_n \frac{|\Gamma_{ms} - \Gamma_{on}|^2}{|1 + \Gamma_{on}|^2(1 - |\Gamma_{ms}|^2)} \\
=& 1.3243 + 4 \times \frac{25}{50} \times \frac{|0.8233\angle - 34^o - 0.61\angle - 39^o|^2}{|1 + 0.61\angle - 39^o|^2(1 - |0.8233\angle - 34^o|^2)} \\
=& 1.3243 + 2.0 \times \frac{0.0489}{2.3202 \times 0.3222} \\
=& 1.3243 + 2.0 \times 0.0654 = 1.4551
\end{aligned}
$$

which is equal to $1.62dB$.

Which corresponds with our assessment based on the noise figure circles in figure 7.

# Design Example 2 : BFU730F SeGe BJT

Problem : You are required to design an 18GHz low noise amplifier having a gain of at least 10dB and a noise figure of less than 2dB, using the BFU730F Silicon-Germainum BJT from NXP. The S-parameters and noise parameters of the device with bias conditions $V_C = 2.0V$, $I_C = 10mA$ are as follows :
S-parameters :

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0.691\angle 63^o & 0.178\angle -20^o \\ 2.108\angle -55^o & 0.218\angle 97^o \end{bmatrix}$$

Noise parameters :

$$F_{min} = 1.79dB$$
$$\Gamma_{on} = 0.667\angle 307^o$$
$$R_n = 28.6\Omega$$

# Design Example 2 : BFU730F SeGe BJT

Solution : Firstly, we need to investigate the stability of the device, for which we will use the Edwards Sinsky stability criteria, i.e.:

$$\mu_1 = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} = \frac{0.4224}{0.3134} = 0.885$$

$$\mu_2 = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta S_{22}^*| + |S_{12}S_{21}|} = \frac{0.8319}{0.8246} = 0.962$$

Since both $\mu_1$ and $\mu_2$ are less than 1 we conclude that the device is potentially unstable. We therefore need to plot stability circles in order to determine the acceptable range of source terminations.

# Design Example 2 : BFU730F SeGe BJT

Since we need to focus on matching the input port to achieve the desired noise specification, we firstly use equations (**??**) and (**??**) to calculate the centre and radius of the source plane stability circle, as follows:

$$C_{SS} = \frac{C_1^*}{|S_{11}|^2 - |\Delta|^2} = \frac{0.6149\angle - 62^o}{0.2492} = 2.4680\angle - 69^o$$

$$r_{SS} = \left| \frac{|S_{12}S_{21}|}{|S_{11}|^2 - |\Delta|^2} \right| = \frac{0.3752}{0.2492} = 1.50598$$

# Design Example 2 : BFU730F SeGe BJT

Once again, we calculate the parameter $N_i$, as defined by (37), for various values of noise figure circle (say, $F = 2dB$, $F = 3dB$ and $F = 5dB$).

We then calculate the respective noise circle centres and radii using (40) and (41). The resulting calculations are summarised in the following table :

| F (dB) | $N_i$ | $\lvert C_{Sn} \rvert$ | $\angle C_{Sn}$ | $\gamma_{Sn}$ |
|---|---|---|---|---|
| 2 | 0.0735 | 0.6213 | $-53^o$ | 0.2002 |
| 3 | 0.4766 | 0.4517 | $-53^o$ | 0.4749 |
| 5 | 1.6231 | 0.2543 | $-53^o$ | 0.7168 |

The above noise figure circles are plotted on the source plane Smith chart, together with the stability circle as shown in figure 8.

# Design Example 2 : BFU730F SeGe BJT

We now check The gain available from the device when terminated for minimum noise figure, i.e. when the source termination is $\Gamma_{on} = 0.667\angle 307^o$. For this we use equation (**??**) for available power gain :

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_{on}|^2)}{|1-S_{11}\Gamma_{on}|^2 - |S_{22}-\Delta\Gamma_{on}|^2}$$

$$= \frac{2.108^2 \times (1-0.667^2)}{|1-0.691\angle 63^o \times 0.667\angle 307^o|^2 - |0.218\angle 97^o - 0.4778\angle 120^o \times 0.667\angle 307^o|^2}$$

$$= \frac{1.1702}{0.2758} = 4.24$$

which is equal to around $6.3dB$. If we set the source termination to obtain minimum noise figure, therefore, we will not be able to achieve the required gain specification. In order to determine a range of source terminations that can achieve the lowest noise figure consistent with 10 dB of gain we should draw the 10 dB constant available gain circle on the source plane and see where this circle intersects with the noise figure circles. The available gain circle are calculated by applying (**??**) and (**??**). Firstly, we need to calculate the normalised gain parameter $g_a$ as defined by (**??**) :

$$g_a = \frac{G_A}{|S_{21}|^2} = \frac{10^{(10/10)}}{2.108^2} = 2.250$$

# Design Example 2 : BFU730F SeGe BJT

We now calculate the centres and radii of the 10 dB constant gain circle on the source reflection coefficient plane, as follows:

$$C_{gS} = \frac{g_a C_1^*}{1 + g_a D_1} = \frac{2.250 \times 0.6149\angle -69.2^o}{1 + 2.250 \times 0.2492} = \frac{1.384\angle -69.2^o}{1.561}$$
$$= 0.8860\angle -69.2^o$$

$$\gamma_{gS} = \frac{\sqrt{1 - 2K|S_{12}S_{21}|g_a + |S_{12}S_{21}|^2 g_a^2}}{1 + g_a D_1} = \frac{0.361}{1.561} = 0.231$$

Where :

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.478\angle 120^o$$
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{21}S_{12}|} = 0.9372$$

$$C_1 = S_{11} - \Delta S_{22}^* = 0.6149\angle 69.2^o$$
$$D_1 = (|S_{11}|^2 - |\Delta|^2) = 0.2492$$

# Design Example 2 : BFU730F SeGe BJT

We can see from figure 8 that there is a region where the 10dB constant gain circle overlaps the $F = 2dB$ constant noise figure circle. Any source termination lying within this region will have a gain greater than 10dB and a noise figure less than 2dB.

We therefore choose a source termination of $\Gamma_S = 0.74\angle - 62^o$ as indicated in figure 8, and we can be confident that this source termination will result in $F < 2dB$ and $G_A > 10dB$.

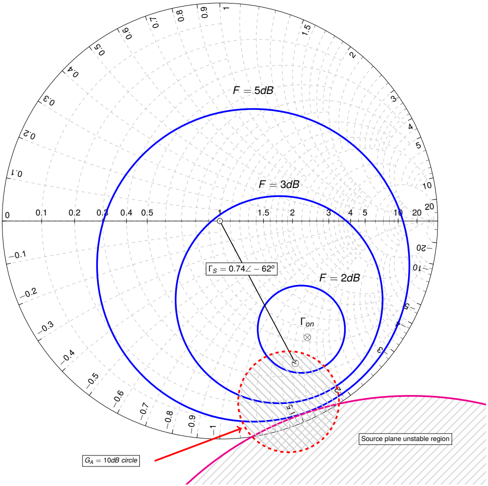



Figure 8 : Constant noise figure circles, $\Gamma_{on}$ and stability and gain circles on the source plane for the NXP BFU730F at 18GHz ($V_C = 2.0V$, $I_C = 10mA$)

# Design Example 2 : BFU730F SeGe BJT

With the chosen source termination of $\Gamma_S = 0.74\angle - 62^o$, the output reflection coefficient of the transistor can be calculated using (**??**), as follows:

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

$$= 0.218\angle 97^o + \frac{0.178\angle - 20^o \times 2.108\angle - 55^o \times 0.74\angle - 62^o}{1 - 0.691\angle 63^o \times 0.74\angle - 62^o}$$

$$= 0.218\angle 97^o + \frac{0.2775\angle - 137^o}{0.4892\angle - 1^o}$$
$$= 0.4694\angle - 157.7^o$$

# Design Example 2 : BFU730F SeGe BJT

We now need to check whether the required value of load termination, set by $\Gamma_L = \Gamma_{out}^*$ is within the load plane stable region. The centre and radius of the load plane stability circle are calculated using (**??**) and (**??**) as follows:

$$C_{SL} = \frac{C_2^*}{|S_{22}|^2 - |\Delta|^2} = \frac{0.2152\angle - 164^o}{-0.1808} = 1.1903\angle - 16.3^o$$

$$r_{SL} = \left| \frac{|S_{12}S_{21}|}{|S_{22}|^2 - |\Delta|^2} \right| = \frac{0.3752}{0.1808} = 2.0753$$

Where : $C_2 = S_{22} - \Delta S_{11}^* = 0.2152\angle - 164^o$

# Design Example 2 : BFU730F SeGe BJT

- The load plane stability circle is plotted in figure 9 together with $\Gamma_L = 0.4694\angle 157.7^o$.
- Since the load plane stability circle encloses the origin, the stable region is represented by the interior the circle. This means that the stable region encompasses most of the load plane Smith Chart except for a small sliver on the left hand side, as shown in figure 9.
- Our chosen value of $\Gamma_L$ is therefore comfortably inside the stable region of the load plane.

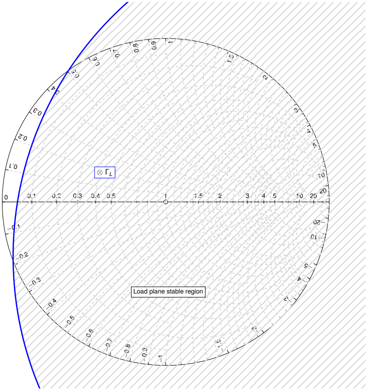



Figure 9 : Load plane stability circles for the NXP BFU730F at 18GHz ($V_C = 2.0V$, $I_C = 10mA$) with designated input match

# Noise factor of passive two-ports

By definition, a passive two-port has a gain, $G$, that is less than unity. Passive circuits are therefore usually characterised by their *attenuation*, which is defined by :

$$A = \frac{1}{G} \tag{42}$$

The equivalent noise temperature of a passive two-port having an attenuation, $A$, and at operating temperature, $T_o$, can be shown to be[2]:

$$T_e = (A - 1)T_o \tag{43}$$

Thus, we can write the output noise temperature of such a passive two-port as:

$$T_{out} = G(T_{in} + T_e) = \frac{(T_{in} + T_e)}{A} = \frac{T_{in}}{A} + \frac{(A-1)T_o}{A} \tag{44}$$

Which reduces to:

$$T_{out} = \frac{T_{in}}{A} - \frac{T_o}{A} + T_o \tag{45}$$

# Noise factor of passive two-ports

The meaning of (45) is as follows:

- as the attenuation, $A$, approaches unity (i.e. the lossless case), we find that $T_{out}$ approaches $T_{in}$. In other words the noise passes through a lossless device unaltered, and the device will generate no internal noise of its own. This makes sense from a physical point of view, since no loss means no internal resistive elements inside the two-port to generate thermal noise.
- Let us now consider the case where the attenuation, $A$, becomes very large. In this case the input noise is completely absorbed by the two-port.
- The noise at the device output consists of noise that is entirely generated inside the two-port. The output noise temperature will therefore become $T_{out} = T_o$ i.e. equal to the physical temperature of the device.

We can now determine the noise factor of a passive two-port by combining (16) on page 14 and (43) to give:

$$F = 1 + \frac{(A-1)T_o}{T_o} = 1 + (A-1) = A \tag{46}$$

In other words, for any passive two-port device, the noise factor, $F$, is equal to the attenuation of the device, $A$.

# Multi-stage low noise amplifier design

- We have seen that a required gain and bandwidth can be obtained by cascading several single stages. In the context of this chapter, cascading stages in this way raises the question of the relationship between the noise factor of a multi-stage amplifier and the noise factors of the individual stages.
- One might intuitively expect that a minimum noise figure multi-stage amplifier could be constructed by simply cascading a number of individual stages each optimised for minimum noise factor. It turns out, however, that this approach does not result in the lowest overall noise figure for the cascade, due to the trade-off between noise factor and gain inherent in single stage amplifier design, as outlined in the previous sections.
- Figure 10 illustrates a cascade of single stage amplifiers, with the $n^{th}$ stage having a noise factor $F_n$ and available gain $G_n$.

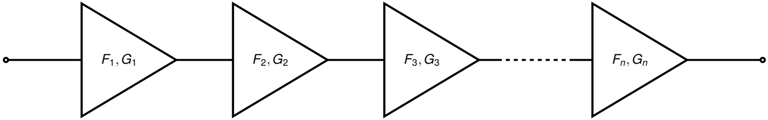



Figure 10 : Cascaded amplifiers

# Multi-stage low noise amplifier design

It turns out that the overall noise factor of the multi-stage amplifier in figure 10 depends not only on the noise factor of the individual stages but also on the gain of all but the first stage. This is embodied in the so called *Friis noise formula* which is named after its originator, Harald Friis[6], and can be stated as follows:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots \frac{F_n - 1}{G_1 G_2 G_3 \ldots G_{n-1}} \tag{47}$$

Where :

$$F = \text{noise factor of the cascade}$$

$$F_n = \text{noise factor of the } n^{th} \text{ stage}$$

$$G_n = \text{gain of the } n^{th} \text{ stage}$$

We can deduce the following by studying equation (47):

(i) The noise factor of the first stage is much more important than the noise factors of subsequent stages, as these are divided by the gain of the preceding stages. This suggests that the first stage noise factor should be made as small as possible.

(ii) In order to make subsequent stage noise factors insignificant, the first stage gain should be as high as possible.

# Multi-stage low noise amplifier design

Consider the case of two stages that are to be cascaded. Let their noise figures be $F_1$ and $F_2$ and their available gains be $G_{a1}$ and $G_{a2}$. There are two possible arrangements as illustrated in figure 11.

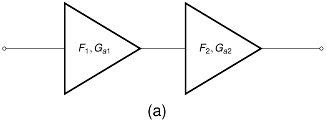



Figure 11 : Two ways of cascading two amplifiers

If stage 1 is placed first, as in figure 11(a), the overall noise factor of the cascade, from equation (47), will be:

$$F_{12} = F_1 + \frac{F_2 - 1}{G_{a1}} \tag{48}$$

On the other hand, if stage 2 is placed first, as in figure 11(b), the overall noise factor of the cascade, from equation (47) will be:

$$F_{21} = F_2 + \frac{F_1 - 1}{G_{a2}} \tag{49}$$

# Multi-stage low noise amplifier design : Noise Measure

In general, one of these possibilities will result in a lower overall noise figure than the other. Suppose that putting stage 1 first results in the lowest overall noise figure, that is:

$$F_{12} < F_{21} \tag{50}$$

Employing equations (48) and (49) results in:

$$F_1 + \frac{F_2 - 1}{G_{a1}} < F_2 + \frac{F_1 - 1}{G_{a2}} \tag{51}$$

Equation (51) can be rearranged to give:

$$\frac{F_1 - 1}{\left(1 - \frac{1}{G_{a1}}\right)} < \frac{F_2 - 1}{\left(1 - \frac{1}{G_{a2}}\right)} \tag{52}$$

Therefore the lowest overall noise figure results from ensuring that the first stage has the lowest value, not of $F$, but of the quantity '$M$' which is defined by :

$$M = \frac{F - 1}{\left(1 - \frac{1}{G_a}\right)} \tag{53}$$

The quantity $M$, which we call the **Noise Measure**, is therefore a more meaningful measure of stage noise performance than noise figure when stages are to be cascaded.

# Multi-stage low noise amplifier design : Noise Measure

If several stages with the same noise measure are cascaded then the noise measure of the cascade will be the same as that of each stage. For such a cascade the overall noise figure is given by [12]:

$$F = M + 1 \tag{54}$$

We can therefore conclude that, in order to build a multi-stage amplifier with the minimum overall noise factor, the first stage, and possibly subsequent stages, should be designed for minimum value of noise measure (i.e. $M_{min}$). We know that noise factor is a function of the source termination alone, so we deduce that the minimum noise measure can be obtained at a particular value of source termination.

We can determine the value of $M_{min}$ and the source termination required to realise it, which we shall refer to as $Y_{om}$ (admittance) or $\Gamma_{om}$ (reflection coefficient) by differentiating equation (53) with respect to the complex source termination ($Y_S$, $\Gamma_S$) and setting the derivatives equal to zero. Alternatively, we can derive circles of constant noise measure in the complex source plane and then consider the circle of zero radius.

# Circles of Constant Noise Measure

As is the case when designing to meet a specific noise factor specification, as covered in section **??**, a graphical representation of the effect of variations in $\Gamma_s$ on the noise measure of an amplifier is a useful design aid.

We will proceed to derive a set of equations for constant noise measure circles based on equation (53). We will employ the available gain equation (**??**) on page **??**, i.e.:

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2-|S_{22}-\Delta\Gamma_S|^2} \quad (\mathbf{??})$$

Substituting equation (**??**) and equation (33), which are both functions only of $\Gamma_S$, into (53) and we have:

$$M = \frac{(F_{min}-1)+4r_n\dfrac{|\Gamma_S-\Gamma_{on}|^2}{|1+\Gamma_{on}|^2(1-|\Gamma_S|^2)}}{\left(1-\dfrac{|1-S_{11}\Gamma_S|^2-|S_{22}-\Delta\Gamma_S|^2}{|S_{21}|^2(1-|\Gamma_S|^2)}\right)} \quad (55)$$

Which can be rearranged as:

$$M = \frac{|S_{21}|^2}{|1+\Gamma_{on}|^2}\cdot\frac{|1+\Gamma_{on}|^2(1-|\Gamma_S|^2)(F_{min}-1)+4r_n|\Gamma_S-\Gamma_{on}|^2}{|S_{21}|^2(1-|\Gamma_S|^2)-|1-S_{11}\Gamma_S|^2+|S_{22}-\Delta\Gamma_S|^2} \quad (56)$$

# Circles of Constant Noise Measure

Expanding out (56) and collecting $\Gamma_S$ terms gives:

$$|\Gamma_S|^2[M|1+\Gamma_{on}|^2(|\Delta|^2-|S_{21}|^2-|S_{11}|^2)-|S_{21}|^2(4r_n-|1+\Gamma_{on}|^2(F_{min}-1))]+$$
$$\Gamma_S(M|1+\Gamma_{on}|^2C_1+4r_n|S_{21}|^2\Gamma_{on}^*)+\Gamma_S^*(M|1+\Gamma_{on}|^2C_1^*+4r_n|S_{21}|^2\Gamma_{on})=$$
$$|S_{21}|^2[|1+\Gamma_{on}|^2(F_{min}-1)+4r_n|\Gamma_{on}|^2]-M|1+\Gamma_{on}|^2(|S_{22}|^2+|S_{21}|^2-1) \quad (57)$$

Where $C_1=S_{11}-S_{22}^*\Delta$.
Equation (57) can be rearranged to give:

$$|\Gamma_S|^2+\Gamma_S\left[\frac{M|1+\Gamma_{on}|^2C_1+4r_n|S_{21}|^2\Gamma_{on}^*}{M|1+\Gamma_{on}|^2(|\Delta|^2-|S_{21}|^2-|S_{11}|^2)-|S_{21}|^2(4r_n-|1+\Gamma_{on}|^2(F_{min}-1))}\right]+$$
$$\Gamma_S^*\left[\frac{M|1+\Gamma_{on}|^2C_1^*+4r_n|S_{21}|^2\Gamma_{on}}{M|1+\Gamma_{on}|^2(|\Delta|^2-|S_{21}|^2-|S_{11}|^2)-|S_{21}|^2(4r_n-|1+\Gamma_{on}|^2(F_{min}-1))}\right]=$$
$$\left[\frac{|S_{21}|^2[|1+\Gamma_{on}|^2(F_{min}-1)+4r_n|\Gamma_{on}|^2]-M|1+\Gamma_{on}|^2(|S_{22}|^2+|S_{21}|^2-1)}{M|1+\Gamma_{on}|^2(|\Delta|^2-|S_{21}|^2-|S_{11}|^2)-|S_{21}|^2(4r_n-|1+\Gamma_{on}|^2(F_{min}-1))}\right] \quad (58)$$

# Circles of Constant Noise Measure

Equation (58) is in the form:

$$|\Gamma_S|^2 + |C_{Sm}|^2 - \Gamma_S^* C_{Sm} - \Gamma_S C_{Sm}^* = \gamma_m^2 \quad (59)$$

which describes a circle in the $\Gamma_S$ plane with centre at $C_{Sm}$ and radius $\gamma_{Sm}$. From (58) we can see that the center of the constant $M$ circle on the $\Gamma_S$ plane is located at:

$$C_{S_m} = \frac{M|1+\Gamma_{on}|^2 C_1^* + 4r_n|S_{21}|^2\Gamma_{on}}{M|1+\Gamma_{on}|^2(|S_{21}|^2+|S_{11}|^2-|\Delta|^2) - |S_{21}|^2(|1+\Gamma_{on}|^2(F_{min}-1)-4r_n)} \quad (60)$$

and the radius are given by:

$$\gamma_{S_m} = \sqrt{\frac{M|1+\Gamma_{on}|^2(1-|S_{22}|^2-|S_{21}|^2) + |S_{21}|^2[|1+\Gamma_{on}|^2(F_{min}-1)+4r_n|\Gamma_{on}|^2]}{M|1+\Gamma_{on}|^2(|\Delta|^2-|S_{21}|^2-|S_{11}|^2) + |S_{21}|^2(|1+\Gamma_{on}|^2(F_{min}-1)-4r_n)} + |C_{S_m}|^2} \quad (61)$$

# Circles of Constant Noise Measure

We can determine the value of the minimum noise measure obtainable with a given device by considering the noise measure circle of zero radius. This means finding a value of $M$ that makes $\gamma_{Sm}$ in (61) equal to zero. This can be done by trial and error.

The source reflection coefficient which gives rise to $M_{min}$ is the centre of the $M_{min}$ noise measure circle. Once the value of $M_{min}$ has been determined, the value of $\Gamma_{om}$ can therefore be determined from equation (60) as:

$$\Gamma_{om} = \frac{M_{min}|1+\Gamma_{on}|^2 C_1^* + 4r_n|S_{21}|^2\Gamma_{on}}{M_{min}|1+\Gamma_{on}|^2(|S_{21}|^2 + |S_{11}|^2 - |\Delta|^2) - |S_{21}|^2(|1+\Gamma_{on}|^2(F_{min}-1) - 4r_n)} \tag{62}$$

With the input port of the transistor terminated in $\Gamma_{om}$, we can calculate the output reflection coefficient looking into the output port of the transistor by employing equation (**??**), i.e.:

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_{om}}{1 - S_{11}\Gamma_{om}} \tag{63}$$

# Design Example 3 : Constant Noise Measure Circles

Problem : Design a single stage amplifier for minimum noise measure using a NE71083 GaAs MESFET at a center frequency of 10GHz and bias conditions $V_{ds} = 3.0V$, $I_d = 8mA$. The S-parameters of the transistor in the common source configuration were measured, with a 50Ω reference impedance, to be as follows:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0.724\angle 46^o & 0.716\angle -47^o \\ 1.303\angle -10^o & 0.616\angle 64^o \end{bmatrix} \quad (64)$$

The following noise parameters were supplied by the manufacturer of the FET:

$F_{min} = 1.7dB$

$\Gamma_{on} = 0.620\angle 148^o$

$r_n = 12\Omega$

# Design Example 3 : Constant Noise Measure Circles

Solution :

The stability of the device is first evaluated using the Edwards Sinsky stability criteria[4] of (**??**) and (**??**), i.e.:

$$\mu_1 = \frac{1-|S_{11}|^2}{|S_{22}-\Delta S_{11}^*| + |S_{12}S_{21}|} = \frac{0.4758}{1.3283} = 0.358 \quad (65)$$

$$\mu_2 = \frac{1-|S_{22}|^2}{|S_{11}-\Delta S_{22}^*| + |S_{12}S_{21}|} = \frac{0.6205}{1.1031} = 0.563 \quad (66)$$

Since both $\mu_1$ and $\mu_2$ are less than unity we conclude that the device is potentially unstable. We therefore need to draw a source plane stability circle to see whether we which source terminations we can use.

# Design Example 3 : Constant Noise Measure Circles

We calculate the centre and radius of the source plane stability circle using equations (**??**) and (**??**) as follows :

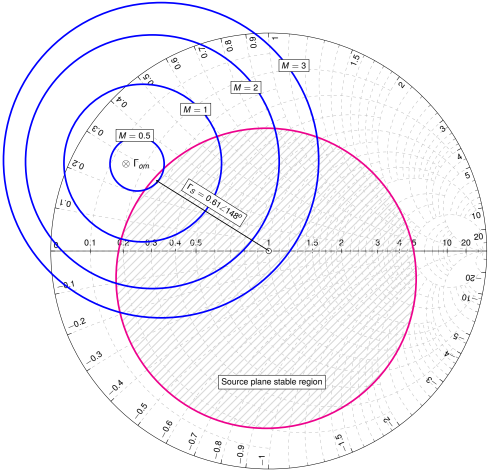
$$C_{SS} = \frac{C_1^*}{|S_{11}|^2 - |\Delta|^2} = \frac{0.1702\angle 85^o}{-1.3559} = 0.126\angle -94^o$$

$$r_{SS} = \frac{|S_{12}S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|} = \frac{0.9329}{1.3559} = 0.688$$

Where : $C_1 = S_{11} - \Delta S_{22}^* = 0.1702\angle -85^o$
By determining the constant noise measure circle of zero radius the minimum noise measure obtainable with this device was found to be $M_{min} = 0.435$. Equation (62) yielded the value of the associated source reflection coefficient, $\Gamma_{om}$, to be $0.729\angle 146.7^o$.

# Design Example 3 : Constant Noise Measure Circles

- Since the stability circle encompasses the origin the stable region is represented by the interior of the stability circle.
- Figure **??** shows that $\Gamma_{om}$ lies outside the stable region in the source reflection coefficient plane. It is therefore not possible to realise a stable amplifier stage having the theoretical minimum noise measure of $M_{min} = 0.435$.
- From figure **??** we can see that the $M = 0.5$ circle just overlaps the source plane stability circle, allowing a small range of $\Gamma_S$ values that will result in a stable amplifier with a value of $M \leq 0.5$. We therefore choose a source termination $\Gamma_S = 0.610\angle 148$ which lies approximately in the centre of this overlapping region, as shown in figure **??**.



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# Noise figure measurement

The Y-factor method involves applying the output of a *noise source* to the input of the DUT and making noise power measurements at the output of the DUT[11]. A conceptual block diagram of a typical noise figure meter is shown in figure 12.

The noise source in figure 12 is powered on and off under the control of a microprocessor inside the instrument. The output signal of the DUT is filtered, downconverted (as necessary, depending on the frequency of operation) and the resulting RMS power level measured and digitised. Each time the noise source is turned on or off the noise power at the output of the of the DUT is thus measured and recorded in the memory of the instrument. The microprocessor carries out the noise figure calculations using the measured data and the equations we will introduce in this section.

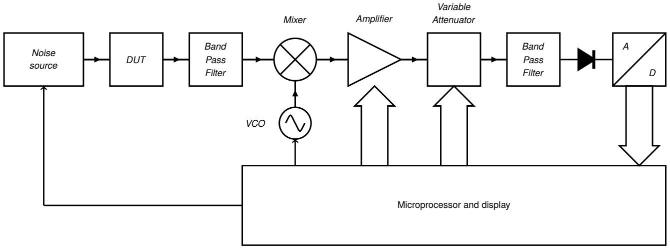


Figure 12 : Noise figure meter simplified block diagram

# Noise figure measurement

The noise source in figure 12 can have two levels of noise output power corresponding to 'cold' and 'hot' noise temperatures ($T_c$ and $T_h$) respectively. In simple terms, these 'hot' and 'cold' temperatures correspond to the noise source having its supply switched on and off[16, 13].

Assuming the DUT is an amplifier, we can define $T_c$ and $T_h$ in terms of the corresponding output noise powers, $N_1$ and $N_2$, of the amplifier in figure 13, i.e.:

$$N_1 = kG\Delta f(T_c + T_a) \tag{67}$$

and

$$N_2 = kG\Delta f(T_h + T_a) \tag{68}$$

Where $G$ is the numerical power gain of the amplifier and $T_a$ is the effective noise temperature of the amplifier.

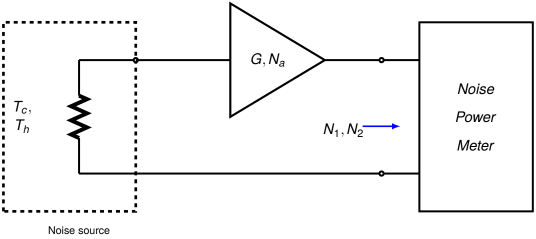


Figure 13 : Amplifier noise model

# Noise figure measurement

- If we measure two noise powers, $N_1$ and $N_2$, at noise temperatures $T_c$ and $T_h$ and plot them on a graph we will get the straight line shown in figure 14.
- The slope of the line is the gain bandwidth product of the amplifier scaled by $k_B$ (i.e. $k_B G \Delta f$).
- The line intercepts the noise power axis at a value *Na*, which corresponds to the equivalent noise power of the amplifier under test, referred to its input.

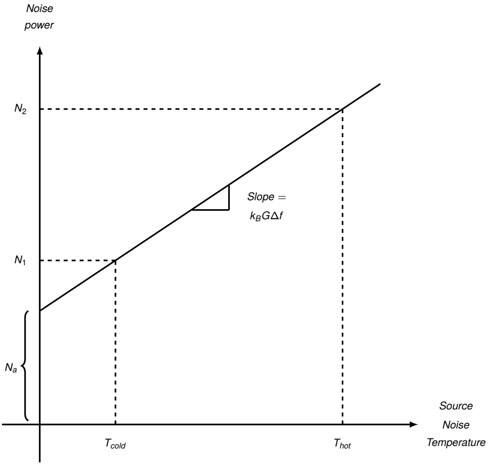


Figure 14 : Effective source temperature versus output noise power

# Noise figure measurement

The so-called 'Y-Factor' is defined as the ratio of 'hot' to 'cold' measured noise powers, as follows[15]:

$$Y = \frac{N_2}{N_1} \tag{69}$$

From (67) and (68) we can write :

$$Y = \frac{T_h + T_a}{T_c + T_a} \tag{70}$$

From (70) we can write $T_a$ in terms of the Y-factor as follows[11]:

$$T_a = \frac{T_h - YT_c}{Y - 1} \tag{71}$$

# Noise figure measurement

The noise factor of the amplifier is related to the effective noise temperature by (15), so we can relate $T_a$ to the system operating temperature as follows :

$$F = \frac{T_a + T_o}{T_o} \tag{72}$$

Combining (71) and (72) we get the noise factor of the amplifier in terms of the Y-factor and the temperatures, $T_o$, $T_c$ and $T_h$ as follows[2]:

$$F = \frac{(T_h/T_o - 1) - (T_c/T_o - 1)}{Y - 1} \tag{73}$$

Note that (73) is independent of the measurement bandwidth, that has been cancelled in the calculation of the Y factor in (70). This is one of the advantages of the Y factor technique. The assumption is often made that $T_c = T_o$, in which case 73 reduces to :

$$F = \frac{(T_h/T_o - 1)}{Y - 1} \tag{74}$$

# Noise figure measurement

Noise sources are usually specified in terms of the *Excess Noise Ratio* (ENR) which is defined as the power level difference between hot and cold states, referenced to the thermal equilibrium noise power at the standard operating temperature, $T_o$. ENR is therefore defined in relation to $T_h, T_c$ and $T_o$ as :

$$ENR = 10\log_{10}\left(\frac{T_h - T_c}{T_o}\right) \tag{75}$$

Again, the assumption is often made that $T_c = T_o$, in which case (75) becomes :

$$ENR(dB) = 10\log_{10}\left(\frac{T_h}{T_o} - 1\right) \tag{76}$$

Considering (74) and (76) we can now write the formula for calculating the noise figure of the DUT, in dB, in terms of the measured Y-factor and the ENR of the source, as follows :

$$F(dB) = ENR(dB) - 10\log_{10}(Y - 1) \tag{77}$$

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