# Lecture 17 - Microwave Mixers

Microwave Active Circuit Analysis and Design

Clive Poole and Izzat Darwazeh

Academic Press Inc.



Lecture 17 - Microwave Mixers

-∢ ≣ →

# **Intended Learning Outcomes**

#### Knowledge

- Understand the role and function of mixers in RF systems.
- Understand various figures of merit used to characterise mixers.
- Understand the strengths and weaknesses of various mixer topologies.
- Skills
  - Be able to design a single balanced diode passive mixer.
  - Be able to design a double balanced diode passive mixer.
  - Be able to design a gilbert cell mixer, based on BJT or FETs.

## **Table of Contents**

Mixer characterisation

**Basic mixer operation** 

Passive mixer circuits

Active mixer circuits

Lecture 17 - Microwave Mixers

э

ヘロト 人間 とくほとくほとう

## **Mixer characterisation**

- Conversion gain
- Isolation
- Dynamic range
- Third-order intercept point
- Noise Figure

・ロト ・聞 と ・ ヨ と ・ ヨ と

# **Conversion gain and isolation**

#### **Conversion gain**

- The conversion gain (or loss) of a mixer is defined as the ratio of the desired output signal power to the input signal power. In the case of a Downconversion mixer the desired output signal is the IF, and the input signal is the RF. In the case of an upconversion mixer the desired output signal is the RF and the input signal is the IF.
- LO power does not feature in conversion gain calculations.
- Active mixers provide a positive conversion gain, whereas passive mixers do not. Conversion gain is also dependent on impedance matching at the input and output ports (i.e. it depends on power transfer at these ports).
- Although IF signal power does not feature in the conversion gain calculation, the level of the LO will affect the conversion gain so this needs to be specified. Conversion gain of a typical active mixer is approximately +10dB, whilst the conversion loss of a typical passive (diode) mixer is approximately -6dB.

#### **Isolation**

 Isolation is a measure of the amount of "leakage" or "feed through" between the mixer ports, especially leakage of the local oscillator signal, as this tends to be the largest of the three.

くロト くぼト くほと くほと

## **Dynamic range**

Dynamic Range (DR) is defined as the range of input power levels for which the output power is linearly proportional to the input power. The proportionality constant is the conversion loss or gain of the mixer.

- The lower bound of dynamic range is set by the noise floor which defines the minimum detectable signal (MDS).
- The 1 dB compression point is commonly used to define the upper bound of of the linear region.
- The system's dynamic range can be defined as the power difference from the MDS to the 1 dB compression point.





A D b 4 A b

# Third-order intercept point

The third order intercept point (or "IP3") is the input (or output) power level at which the nonlinear intermodulation products caused by the third-order non-linearities are equal to the desired signal. Two different definitions for intercept points

are in use:

- <u>Based on harmonics</u>: The device is tested using a single input tone. The nonlinear products caused by n<sup>th</sup> order nonlinearity appear at n times the frequency of the input tone.
- 2. Based on intermodulation products : The device is fed with two sine tones with a small frequency difference. The  $n^{th}$  order intermodulation products then appear at n times the frequency spacing of the input tones. This is the more commonly used approach.







# **Noise Figure**

- The same definitions of noise figure that apply to amplifiers may be applied to mixers, provided we take account of the fact that the noise generated by the mixer will be upconverted or downconverted to a different frequency range
- For passive mixers, where there is no gain, only loss, the noise figure is approximately equal to the insertion loss
- Active mixers typically exhibit higher noise figure than passive mixers at comparable linearity, which is as one would expect from when comparing any active circuit with its passive equivalent[3]

イロト イ押ト イヨト イヨト

## **Table of Contents**

Mixer characterisation

Basic mixer operation

Passive mixer circuits

Active mixer circuits

э

・ロト ・聞 ト ・ ヨト ・ ヨトー

## **Basic mixer operation**





Figure 4 : Single balanced mixer LO switching waveform

Figure 3 : Single balanced mixer equivalent circuit

$$S(v_{LO}) = \begin{cases} 1 & \text{if } v_{LO} \ge 0\\ 0 & \text{if } v_{LO} < 0 \end{cases}$$
(1)

э

## Single balanced mixer output waveform



Figure 5 : Single balanced mixer output waveform

voltage

## **Double balanced mixer operation**





Figure 6 : Double balanced mixer equivalent circuit

Figure 7 : Double balanced mixer LO switching waveform

$$S(v_{LO}) = \begin{cases} 1 & \text{if } v_{LO} \ge 0\\ -1 & \text{if } v_{LO} < 0 \end{cases}$$
(2)

ъ

#### Double balanced mixer output waveform



Figure 8 : Double balanced mixer output waveform

voltage

## Double balanced mixer output waveform

After low pass filtering :



Figure 9 : Double balanced mixer output waveform after filtering

## **Table of Contents**

Mixer characterisation

**Basic mixer operation** 

Passive mixer circuits

Active mixer circuits

Lecture 17 - Microwave Mixers

ヘロト 人間 とくほとく ほとう

## **Non-linear mixer**



Figure 10 : Mixing using a non-linear device

э

・ロト ・聞 と ・ ヨ と ・ ヨ と ・

#### **Non-linear mixer operation**

Given the current/voltage characteristic for a semiconductor diode :

$$I = I_S \left( e^{qV_D/k_B T} - 1 \right) \tag{3}$$

where  $V_D$  is the voltage across the diode, *I* is the forward current and  $I_S$  is the reverse saturation current. We can replace the exponential function  $e^x$  by the following equivalent Taylor series[5]:

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!} \tag{4}$$

For small values of x, (4) can be approximated by :

$$e^x - 1 \approx x + \frac{x^2}{2} \tag{5}$$

The sum of the two input voltage signals  $v_1 + v_2$  is applied to a diode, and the output voltage,  $v_o$  is proportional to the current through the diode. Ignoring DC terms, the output signal voltage will be of the form :

$$v_o = (v_1 + v_2) + \frac{1}{2}(v_1 + v_2)^2 + \dots$$
 (6)

which can be rewritten as :

$$v_o = (v_1 + v_2) + \frac{1}{2}v_1^2 + v_1v_2 + \frac{1}{2}v_2^2 \tag{7}$$

## **Double-balanced diode mixer**



Figure 11 : Double balanced diode mixer

э

## Double-balanced diode mixer : positive L.O. cycle



Figure 12 : Double balanced diode mixer model : positive LO cycle

## Double-balanced diode mixer : negative L.O. cycle



Figure 13 : Double balanced diode mixer model : negative LO cycle

## **Table of Contents**

Mixer characterisation

**Basic mixer operation** 

Passive mixer circuits

Active mixer circuits

Lecture 17 - Microwave Mixers

・ロト ・ 四ト ・ ヨト ・ ヨト ・

# Active mixer building blocks

- The V/I conversion function is usually carried out by a voltage controlled current source.
- The mixing function consists of multiplying one signal by another in the time domain using a current multiplier.
- The I/V conversion function can be carried out simply by a means of a load resistor.



Figure 14 : Active mixer building blocks

VRF

A E > A E >

# Single-balanced active mixer using BJT

- The current source transistor, Q<sub>3</sub>, sets the total current in the upper two transistors, which then compete for a fraction of this current.
- The load resistors, R<sub>L</sub>, convert the collector currents into voltages. The gain of a differential pair depends on the transconductance, g<sub>m</sub>, of the two transistors which, in turn, depends on their collector current (according to g<sub>m</sub> = I<sub>C</sub>/V<sub>T</sub>).
- The circuit of figure 15 can therefore be used as a mixer when the current in Q<sub>3</sub>, is modulated by the RF signal voltage.
- We apply the local oscillator signal as a differential voltage to the bases of Q<sub>1</sub> and Q<sub>2</sub>, and take the IF signal as a differential voltage across the collectors of Q<sub>1</sub> and Q<sub>2</sub>.
- The degenerative feedback resistance, *R<sub>e</sub>*, is added to the current source to improve its linearity.



Figure 15 : Single-balanced active mixer using BJTs

Assuming the two transistors  $Q_1$  and  $Q_2$  in figure 15 have identical voltage-current characteristics, the relationships between the small signal collector currents and base-emitter voltages of  $Q_1$  and  $Q_2$  are as follows :

$$i_{C1} = I_S e^{(v_{BE1}/V_T)}$$
 (8)

$$i_{C2} = I_S \ e^{(v_{BE2}/V_T)} \tag{9}$$

Where  $I_S$  is the saturation current and  $V_T$  is the thermal voltage defined by  $V_T = kT/q \approx 26mV$  at room temperature (around T=290 K). Alternatively we can write :

$$v_{BE1} = V_T \ln\left(\frac{i_{C1}}{I_S}\right) \tag{10}$$

$$v_{BE2} = V_T \ln\left(\frac{i_{C2}}{I_S}\right) \tag{11}$$

The voltage at the common emitter point,  $v_{EE}$ , in figure 15 is given by :

$$v_{EE} = \overline{v_{LO}} - v_{BE1} = v_{LO} - v_{BE2} \tag{12}$$

From (12) we can write :

$$v_{LO} - \overline{v_{LO}} = v_{BE2} - v_{BE1} \tag{13}$$

 $v_{LO} - \overline{v_{LO}}$  is the differential local oscillator input signal voltage,  $\Delta v_{LO}$ , so applying (10) we can write :

$$\Delta v_{LO} = v_{LO} - \overline{v_{LO}} = V_T \ln\left(\frac{i_{C2}}{I_S}\right) - V_T \ln\left(\frac{i_{C1}}{i_S}\right)$$
(14)

$$\Delta v_{LO} = V_T \ln \left(\frac{i_{C2}}{i_{C1}}\right) \tag{15}$$

Therefore :

$$\left(\frac{i_{C1}}{i_{C2}}\right) = e^{(-\Delta v_{LO}/V_T)}$$
(16)

For most microwave transistors the current gain is high, therefore we can usually ignore the base current, i.e.  $i_E \approx i_C$ . Therefore from figure 15 we can write :

$$i_{EE} = i_{C1} + i_{C2} \tag{17}$$

Combining (16) and (17) we can now write the collector signal currents of  $Q_1$  and  $Q_2$  in figure 15 in terms of  $\Delta v_{LO}$  and  $I_{EE}$  as :

$$i_{C1} = \frac{i_{EE}}{1 + e^{(\Delta v_{LO}/V_T)}}$$
(18)

$$i_{C2} = \frac{i_{EE}}{1 + e^{(-\Delta v_{LO}/V_T)}}$$
(19)

The difference between the two collector currents can now be written as follows :

$$\Delta i_{IF_{12}} = i_{C1} - i_{C2} \tag{20}$$

$$= I_{EE} \left( \frac{1}{1 + e^{(\Delta v_{LO}/V_T)}} - \frac{1}{1 + e^{(-\Delta v_{LO}/V_T)}} \right)$$
(21)

Which can be rewritten as :

$$\Delta i_{IF_{12}} = i_{EE} \left( \frac{e^{(-\Delta v_{LO}/2V_T)} - e^{(\Delta v_{LO}/2V_T)}}{e^{(-\Delta v_{LO}/2V_T)} + e^{(\Delta v_{LO}/2V_T)}} \right)$$
(22)

Equation (22) can be more neatly expressed by using the definition of the hyperbolic tangent (tanh) function, which is defined as :

$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$
(23)

We can now rewrite (20) as :

$$\Delta i_{IF_{12}} = i_{EE} \tanh\left(\frac{\Delta v_{LO}}{2V_T}\right) \tag{24}$$

< ⊒ →

Now, if we assume that the bias current  $I_{EE}$  is modulated by  $v_{RF}$ , as illustrated in figure 15, we can replace the simple term  $I_{EE}$  in (24) by  $(I_{EE_o} + g_{m_3}v_{RF})$  where  $I_{EE_o}$  is the quiescent DC bias current and  $g_{m_3}$  is the transconductance of the current source transistor,  $Q_3$ . We can now write :

$$\Delta i_{IF_{12}} = \left(I_{EE_o} + g_{m_3} v_{RF}\right) \tanh\left(\frac{\Delta v_{LO}}{2V_T}\right) \tag{25}$$

Which can be expanded to :

$$\Delta i_{IF_{12}} = I_{EE_o} \tanh\left(\frac{\Delta v_{LO}}{2V_T}\right) + g_{m_3} v_{RF} \tanh\left(\frac{\Delta v_{LO}}{2V_T}\right)$$
(26)

Consider the Maclaurin series expansion of tanh(x), as follows[5]:

$$\tanh(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 \cdots$$
 (27)

From (27) we can show that  $tanh(x) \approx x$  for small values of x (i.e. for values of x up to x = 0.5 we have tanh(x)/x > 0.92). So we can approximate the tanh in (26) on the assumption that  $v_{RF}$  and  $v_{LO}$  are small (i.e. less than  $V_T = 26 \ mV$  at room temperature). We therefore have :

$$\Delta i_{IF_{12}} \approx I_{EE_o} \left( \frac{\Delta v_{LO}}{2V_T} \right) + g_{m_3} \left( \frac{\Delta v_{LO} v_{RF}}{2V_T} \right)$$
(28)

The first term in equation (28) represents the Local Oscillator leakage component, which is proportional to the DC bias current,  $i_{EE_o}$ . The second term contains the product term we are interested in, namely ( $v_{RF} \Delta v_{LO}$ ).

- The deficiencies of the single-balanced mixer can be overcome by adopting a double balanced design, and using a balanced RF signal feed.
- This can be achieved by combining two single balanced circuits, one being driven by v<sub>RF</sub> and the other driven by its inverse, v<sub>RF</sub>. The balanced LO inputs and IF outputs are combined by connecting the respective nodes together.
- The resultant circuit is known as a Gilbert cell[1] the basic topology of which is shown in figure 16.



Figure 16 : Gilbert cell mixer topology

< □ > < 同

The second transistor pair in figure 16,  $Q_4$ ,  $Q_5$ , has a signal response similar to the original pair,  $Q_1$ ,  $Q_2$ , but  $180^\circ$  out of phase, as this pair is being driven by an inverted version of the LO signal (relative to that driving  $Q_1$  and  $Q_2$ ). By analogy with equation (28) we can therefore write:

$$\overline{\Delta i_{IF_{45}}} \approx I_{EE} \left( \frac{\Delta v_{LO}}{2V_T} \right) - g_{m_6} v_{RF} \left( \frac{\Delta v_{LO}}{2V_T} \right)$$
(29)

Where, in this case,  $\Delta i_{IF_{45}} = i_{C4} - i_{C5}$ 

If we ensure that all the transistors being used have identical characteristics, so the  $g_m$  values are the same in all cases and specifically  $g_{m_3} = g_{m_6}$ , the differential IF output current of the circuit of figure 16 is given by:

$$\Delta i_{IF} = \Delta i_{IF_{12}} - \overline{\Delta i_{IF_{45}}} \tag{30}$$

$$\approx 2g_m v_{RF} \left(\frac{\Delta v_{LO}}{2V_T}\right) = \left(\frac{g_m}{V_T}\right) v_{LO} \cdot v_{RF} \tag{31}$$

The above subtraction of  $\overline{i}_{IF_{45}}$  from  $i_{IF_{12}}$  has the effect, to a first order approximation, of cancelling the common term containing the unmodulated  $v_{LO}$  signal, leaving only the desired product term  $v_{LO} \cdot v_{RF}$ . Thus, the IF feed-through component that was present at the output of the single balanced mixer described by (26) has been removed. This is the primary benefit of using the double balanced topology in figure 16.

イロト 不得 トイヨト イヨト 三日

- Figure 17 shows a practical Gilbert Cell implementation such as would be implemented in MMIC form. The common bias current, *I<sub>EE</sub>*, is set by the fixed current source *Q*<sub>7</sub>.
- The gain of the two differential amplifiers, formed of Q<sub>1</sub> & Q<sub>2</sub> and Q<sub>4</sub> & Q<sub>5</sub>, is controlled by modulating the emitter bias current via the transistors Q<sub>3</sub> and Q<sub>6</sub>.
- In normal operation the Local Oscillator signal is applied differentially to the bases of Q<sub>1</sub> and Q<sub>5</sub> (positive phase) and Q<sub>2</sub> and Q<sub>4</sub> (negative phase), whilst the RF signal is applied differentially to the bases of Q<sub>3</sub> (positive phase) and Q<sub>6</sub> (negative phase).
- The IF signal is taken differentially from the collectors of the two upper transistor pairs.
- Another enhancement applied to the Gilbert Cell in figure 17 is the addition of the emitter degeneration resistors, R<sub>E</sub>. These are used in practical circuits to improve linearity at the expense of some conversion gain[2].
- The effect of  $R_E$  is to reduce the transconductance of the lower transistors,  $Q_3$  and  $Q_6$  by a factor of  $1/(1 + g_m R_E)$ , due to the action of local feedback.

ヘロト 人間 とくほ とくほ とう



Figure 17 : Gilbert cell mixer implementation using BJT >> < =>

Lecture 17 - Microwave Mixers

∢ 臣 ≯

The collector signal currents of the upper pair transistors illustrated in figure 17, once again ignoring DC bias and base currents, are as follows :

$$i_{C1} = \frac{i_{C3}}{1 + e^{-v_{LO}/V_T}}$$
(32)

$$i_{C2} = \frac{i_{C3}}{1 + e^{\nu_{LO}/V_T}}$$
(33)

$$i_{C4} = \frac{i_{C6}}{1 + e^{\nu_{LO}/V_T}} \tag{34}$$

$$i_{C5} = \frac{i_{C6}}{1 + e^{-v_{LO}/V_T}}$$
(35)

For the lower pair transistors,  $Q_3$  and  $Q_6$ , we can write :

$$i_{C3} = \frac{I_{EE}}{1 + e^{-v_{RF}/V_T}}$$
(36)

$$i_{C6} = \frac{I_{EE}}{1 + e^{v_{BF}/V_T}}$$
(37)

< ⊒⇒

Noting that, in the case of the lower transistors,  $I_{EE}$  is the fixed bias current provided by  $Q_7$ . Combining (32) through (37), we obtain expressions for the collector currents  $I_{C1}$ ,  $I_{C2}$ ,  $I_{C4}$ , and  $I_{C5}$  in terms of input signal voltages  $v_{RF}$  and  $v_{LO}$ .

$$i_{C1} = \frac{I_{EE}}{\left[1 + e^{-\nu_{LO}/V_T}\right] \left[1 + e^{-\nu_{RF}/V_T}\right]}$$
(38)

$$i_{C2} = \frac{I_{EE}}{[1 + e^{-v_{RF}/V_T}] [1 + e^{v_{LO}/V_T}]}$$
(39)

$$i_{C4} = \frac{I_{EE}}{\left[1 + e^{\nu_{LO}/V_T}\right] \left[1 + e^{\nu_{RF}/V_T}\right]}$$
(40)

$$i_{C5} = \frac{I_{EE}}{\left[1 + e^{\nu_{RF}/V_{T}}\right] \left[1 + e^{-\nu_{LO}/V_{T}}\right]}$$
(41)

イロト イ押ト イヨト イヨト

With reference to figure 17 the differential output current is given by :

$$\Delta i_{IF} = (i_{C1} + i_{C4}) - (i_{C2} + i_{C5}) \tag{42}$$

Which can be rearranged as :

$$\Delta i_{IF} = (i_{C1} - i_{C5}) - (i_{C2} - i_{C4}) \tag{43}$$

Applying equations (38) to (43) and using the definition of tanh, we can rewrite (43) as :

$$\Delta i_{IF} = I_{EE} \tanh\left(\frac{v_{LO}}{2V_T}\right) \cdot \tanh\left(\frac{v_{RF}}{2V_T}\right) \tag{44}$$

The differential output current is converted into a differential voltage by the load resistors,  $R_L$ , so we can write :

$$v_{IF} = I_{EE}R_L \tanh\left(\frac{v_{LO}}{2V_T}\right) \cdot \tanh\left(\frac{v_{RF}}{2V_T}\right)$$
 (45)

< ⊒ → -

If we again assume that both  $v_{RF}$  and  $v_{LO}$  are small (i.e. less than  $V_T$ ), we can apply the approximation  $tanh(x) \approx x$  to (45) and thereby obtain:

$$v_{IF} \approx \left(\frac{I_{EE}R_L}{4V_T^2}\right) v_{LO} v_{RF} \tag{46}$$

Equation (46) means that, for small enough signals, the differential output IF voltage is directly proportional to the product of the RF and LO input signal voltages. Equation (46) is true irrespective of the polarities of  $v_{RF}$  and  $v_{LO}$ .

The output of a Gilbert cell is therefore a true four quadrant multiplication of the differential base voltages of the LO and RF inputs. For this reason the circuit, although applied as a mixer in this context, is often simply referred to as a *Gilbert cell multiplier*.

## **Gilbert Cell Operating Modes**

In practice there are three distinct operating modes for the Gilbert cell, according to the magnitudes of  $v_{RF}$  and  $v_{LO}$  relative to  $V_T$ , as follows :

- If both v<sub>RF</sub> and v<sub>LO</sub> are much less than V<sub>T</sub>, then the hyperbolic tangent function is approximately linear and the circuit behaves as a true four quadrant analogue voltage multiplier, as per (46). The input voltage range can be extended by adding 'predistortion' circuits at the inputs, which have an approximately tanh<sup>-1</sup> characteristic. This technique is sometimes used at lower frequencies but is not common at microwave frequencies[1].
- 2. If one of the input voltages significantly exceeds V<sub>T</sub> then one of the transistor pairs will be driven into saturation and will behave like on/off switches. This is effectively equivalent to multiplying the other input signal by a square wave. This mode of operation is quite common in downconversion mixers where the IF signal is a lot larger than the RF signal. The 'squaring' of the LO signal in this mode is not a problem as the information content of the RF signal is preserved.
- 3. If both of the input voltages significantly exceed V<sub>7</sub> then all of the transistors are operating as switches. This mode is sometimes employed when the mixer is being used as a phase detector, as the phase relationship between the two input signals is preserved, even though any information contained in the signal amplitudes will have been lost.

ヘロト ヘ戸ト ヘヨト ヘヨト

### **Conversion gain of the Gilbert Cell**

Consider (45) for the case of a downconversion mixer, where the RF signal is of a small amplitude. Using the approximation  $tanh(x) \approx x$  for  $x \ll 1$ , we can write :

$$v_{IF} = \frac{I_{EE}R_L v_{RF}}{2V_T} \tanh\left(\frac{v_{LO}}{2V_T}\right)$$
(47)

Where  $I_{EE}$  is the DC bias current through  $Q_7$ . We will simplify the analysis by firstly considering the absence of emitter degeneration resistors, i.e. we set  $R_E = 0$ . We can then define the transconductance of the bias transistor,  $Q_7$ , in figure 17 as :

$$g_{m_7} = \frac{I_{EE}}{V_T} \tag{48}$$

We now substitute (48) into (47) to give :

$$v_{IF} = \tanh\left(\frac{v_{LO}}{2V_T}\right)\frac{g_{m_7}R_Lv_{RF}}{2}$$
(49)

which gives the conversion gain of the circuit of figure 17 as :

$$\frac{v_{IF}}{v_{RF}} = \tanh\left(\frac{v_{LO}}{2V_T}\right)\frac{g_{m_7}R_L}{2}$$
(50)

## **Conversion gain of the Gilbert Cell**

For a downconversion mixer we expect the RF input to be of small amplitude and we need its treatment to be as linear as possible, so as to preserve the information content[4]. The handling of the LO input, on the other hand, need not be linear, since the LO is of known amplitude and frequency and therefore has no information content. Distortion of the LO signal is of no consequence, and so we may choose the LO signal amplitude so as to maximise conversion efficiency. In fact, the LO input is usually designed to switch the upper transistor quad so that for half the cycle  $Q_1$  and  $Q_5$  are 'on' and taking all of the current  $i_{C3}$  and  $i_{C6}$ . For the other half of the LO cycle,  $Q_1$  and  $Q_5$  are 'off' and  $Q_2$  and  $Q_4$  are on, so all of  $i_{C3}$  and  $i_{C6}$  flows through these respective transistors. In other words, for a switching Gilbert Cell where  $v_{LO} >> 2V_T$ , then (50) can be approximated as :

$$\frac{v_{IF}}{v_{RF}} = u(v_{LO}) \left(\frac{g_m R_L}{2}\right)$$
(51)

where

$$u(v_{LO}) = \begin{cases} 1 & \text{if } v_{LO} \ge 0\\ -1 & \text{if } v_{LO} < 0 \end{cases}$$
(52)

We can represent the square wave function  $u(v_{LO})$  by its Fourier series expansion:

$$u(v_{LO}) = \frac{4}{\pi} \left( \sin(\omega_{LO}t) + \frac{1}{3}\sin(3\omega_{LO}t) + \frac{1}{5}\sin(5\omega_{LO}t) + \cdots \right)$$
(53)

- < ⊒ → -

## **Conversion gain of the Gilbert Cell**

The conversion gain of a downconversion mixer is defined as the ratio of IF signal amplitude to RF signal amplitude. We are therefore only interested in the first term in (53), which has an amplitude of  $\pi/4$ . The conversion gain given by (51) now becomes :

$$\frac{v_{RF}}{v_{IF}} \approx \frac{2}{\pi} g_m R_L \tag{54}$$

We now consider the effect of finite values of  $R_E$ , which has the effect of reducing the transconductance of the lower transistors,  $Q_3$  and  $Q_6$  by a factor of  $(1 + g_m R_E)$ . With finite  $R_E$  the conversion gain given by (50) now becomes :

$$\frac{v_{IF}}{v_{RF}} = -\tanh\left(\frac{v_{LO}}{2V_T}\right)\left(\frac{g_m R_L}{1 + g_m R_E}\right)$$
(55)

Employing the function  $u(v_{LO})$  defined above yields:

$$\frac{v_{IF}}{v_{RF}} = u(v_{LO}) \left(\frac{g_m R_L}{1 + g_m R_E}\right)$$
(56)

The voltage gain of the downconversion mixer with the emitter degeneration resistors added is given by :

$$\frac{V_{RF}}{V_{IF}} \approx \frac{2}{\pi} \left( \frac{g_m R_L}{1 + g_m R_E} \right)$$
(57)

## **The FET Gilbert Cell**



< • • • • •

## References

#### B. Gilbert.

The multi-tanh principle: a tutorial overview.

Solid-State Circuits, IEEE Journal of, 33(1):2-17, January 1998.



Rok-Hee Lee, Jae-Young Lee, Sang-Hun Lee, B. Shrestha, Sun-Jin Kim, G.P. Kennedy, Chan Hyeong Park, Nam-Young Kim, and Sang-Hoon Cheon.

Circuit techniques to improve the linearity of an up-conversion double balanced mixer with an active balun using InGaP/GaAs HBT technology.

In Microwave Conference Proceedings, 2005. APMC 2005. Asia-Pacific Conference Proceedings, volume 2, pages 4 pp.–, December 2005.



S.A. Maas.

#### Microwave mixers.

The Artech House Microwave Library. Artech House, Incorporated, 1993.



#### J.W.M. Rogers and C. Plett.

Radio Frequency Integrated Circuit Design.

Artech House Microwave Library. Artech House, Incorporated, 2014.



K.A. Stroud and D.J. Booth.

Advanced Engineering Mathematics.

Palgrave Macmillan Limited, 2011.