Lecture 3 - Practical Transmission Lines

Microwave Active Circuit Analysis and Design

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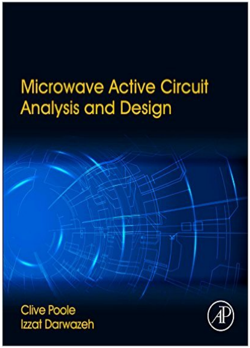

Intended Learning Outcomes

- Knowledge
  - Be familiar with the various types of practical transmission lines, their construction, properties and applications.
  - Be familiar with the various modes of electromagnetic propagation in practical transmission line structures.
  - Understand the strengths and weaknesses of various practical transmission line structures.
  - Be familiar with several common microstrip discontinuities and their effects.
- Skills
  - Be able to calculate the cut-off frequency of rectangular and circular waveguide of specified dimensions.
  - Be able to calculate the characteristic impedance of a co-axial cable of specified dimensions.
  - Be able to calculate the characteristic impedance of a microstrip line of specified dimensions (analysis) and design a microstrip line of specified characteristic impedance (synthesis).
  - Be able to model various common microstrip discontinuities as equivalent circuits or two-port networks.

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Waveguide

- Waveguide is a form of transmission line which has only one outer conducting surface.
- Energy is transmitted in the form of electric and magnetic fields which are confined within the waveguide
- The physical dimensions of the waveguide are determined by the frequency of operation :

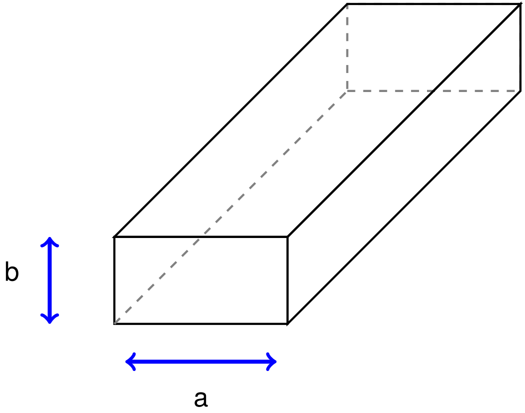
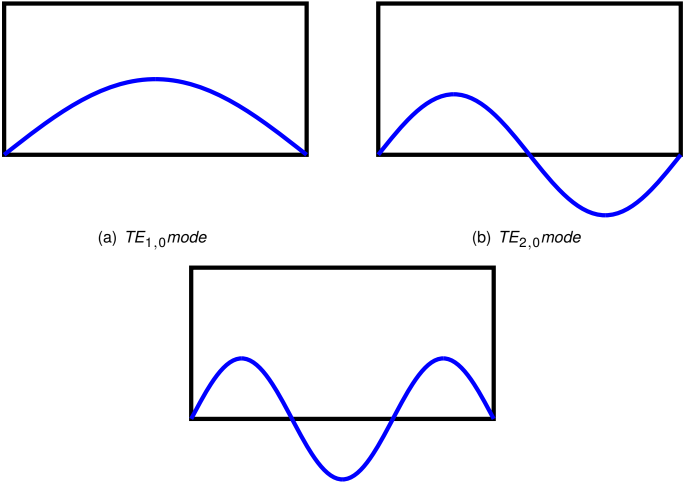


Figure 1 : Rectangular waveguide

Waveguide



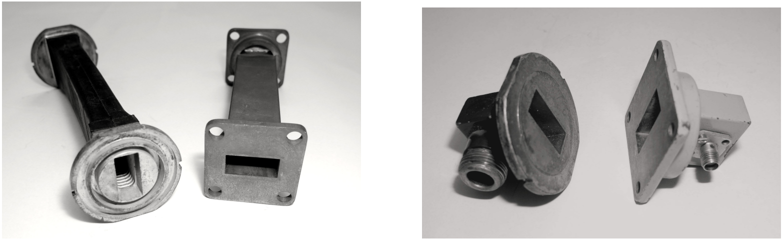
(a) $TE_{1,0} mode$

(b) $TE_{2,0} mode$

(c) $TE_{3,0} mode$

Figure 2 : TE modes for a rectangular waveguide

Waveguide



(a) (b)

Figure 3 : Examples of rectangular waveguide sections and waveguide to co-axial transitions (Reproduced by kind permission of University College London, Department of Electronic and Electrical Engineering)

Waveguide

The cut-off frequency of a rectangular waveguide for a given *TE* mode is given by[6] :

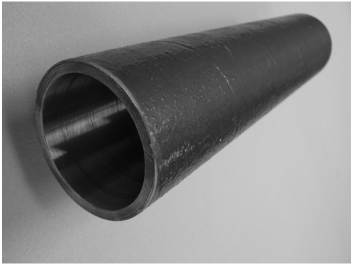
$$(\omega_c)_{m,n} = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{1}$$

where :
a = Inside width (longest dimension) in metres.
b = Inside height (shortest dimension) in metres.
m = Number of half-wavelength variations of the field in the 'a' direction
n = Number of half-wavelength variations of the field in the 'b' direction
ε = Permittivity ($8.854187817 \times 10^{-12}$ for free space)
μ = Permeability ($4\pi \times 10^{-7}$ for free space)
For an air-filled rectangular waveguide (the most common type) we can simplify (1) to :

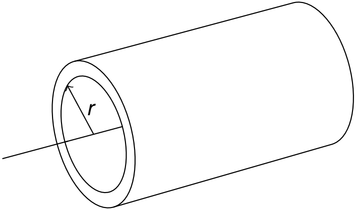
$$(f_c)_{m,n} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \tag{2}$$

Where c is the speed of light in air ($\approx 3 \times 10^8 m/s$).

Circular Waveguide



(a)



(b)

Figure 4 : Circular waveguide (photo reproduced by kind permission of University College London, Department of Electronic and Electrical Engineering)

Circular Waveguide

The lower cut-off wavelength for a particular $TE_{m,n}$ mode in circular waveguide of internal radius 'r', as defined in figure 4(b), is determined by[6]:

$$\lambda_c = \frac{2\pi r}{k'_{m,n}} \qquad (3)$$

Where $k'_{m,n}$ are the roots of the electric wave equation for circular waveguide, as listed in the following table:

Table 1 : Coefficients for circular waveguide TE Modes

m	$k'_{m,1}$	$k'_{m,2}$	$k'_{m,3}$
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

The lower cut-off wavelength for a particular $TM_{m,n}$ mode in circular waveguide of radius 'r' is determined by :

$$\lambda_c = \frac{2\pi r}{k_{m,n}} \qquad (4)$$

Where $k_{m,n}$ are the roots of the magnetic wave equation for circular waveguide, as listed in the following table:

Table 2 : Coefficients for circular waveguide TM Modes

m	$k_{m,1}$	$k_{m,2}$	$k_{m,3}$
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

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Co-axial cable

For the co-axial cable of figure 5 the shunt capacitance per unit length, in farads per metre is given by[1]:

$$C = \frac{2\pi\varepsilon}{\ln(D/d)} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(D/d)} \tag{5}$$

Where D is the outer conductor internal diameter and d is the inner conductor diameter. Series inductance per unit length, in Henrys per metre is given by[1]:

$$L = \frac{\mu}{2\pi}\ln\left(\frac{D}{d}\right) = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{D}{d}\right) \tag{6}$$

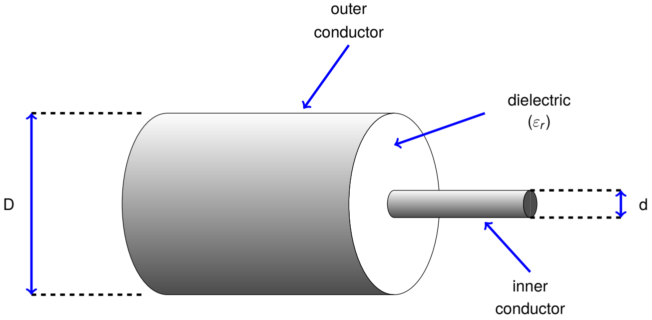

Figure 5 : Co-axial cable construction

Characteristic impedance of co-axial cable

Neglecting resistance per unit length (a reasonable assumption for most practical cables), the characteristic impedance is determined from the capacitance per unit length, given by (5), and the inductance per unit length, given by (6), by applying expression (**??**) for a generic transmission line as follows :

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} \ln\left(\frac{D}{d}\right) \tag{7}$$

$$\boxed{\approx \frac{138\Omega}{\sqrt{\varepsilon_r}} \log_{10}\left(\frac{D}{d}\right)}$$

- The loss per unit length is a combination of the loss in the dielectric material filling the cable, and resistive losses in the center conductor and outer shield. These losses are frequency dependent, the losses becoming higher as the frequency increases.
- Skin effect losses in the conductors can be reduced by increasing the diameter of the cable.

The velocity of propagation inside the cable depends on the dielectric constant and permeability (which is usually 1), i.e. :

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}} \tag{8}$$

Semi-rigid Co-axial cable

In the case of flexible co-axial cables, the inner conductor is usually made from multi-stranded wire and the outer conductor is made from wire braid. At microwave frequencies, we often come across "rigid" or "semi-rigid" co-axial cables where both inner and outer conductors are fabricated as solid metal cylinders, common materials being tinned or silver plated copper, copper clad steel and copper clad aluminium.

The advantage of rigid and semi-rigid cables is that they generally have lower losses than flexible co-ax cables and are generally used in applications where flexibility is not so important, such as fixed interconnections between subsystems inside pieces of equipment, for example. Some sections of semi-rigid co-ax, with typical RF connectors, are shown in figure 6.

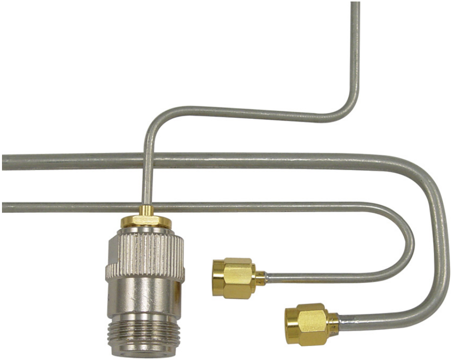

Figure 6 : A selection of semi-rigid co-axial cables, with RF connectors (Reproduced by kind permission of L-com Global Connectivity, 45 Beechwood Dr.N. Andover, MA, USA (www.l-com.com))

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Twisted pair

The characteristic impedance of the twisted pair illustrated in figure 7(b) can be calculated as follows.

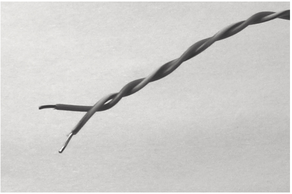
$$Z_o = \frac{120}{\sqrt{\varepsilon_r}} \cdot \ln\left[\frac{D}{r}\right] \qquad (9)$$

Where :

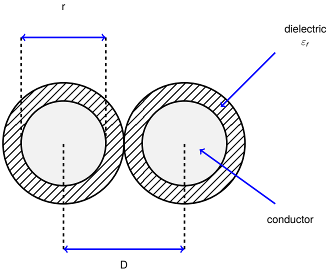
D =the distance between the two conductors (centre to centre).

r =radius of the conductors.

ε_r =effective dielectric constant of the insulating material.



(a)



(b)

Figure 7 : Twisted pair cable and cross-section

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Microstrip

- Microstrip is now widely used because it is compatible with modern component packaging and can be fabricated using standard PCB fabrication techniques.
- A typical microstrip transmission line consists of a narrow strip of conducting material, of width w, separated from a ground plane by a dielectric substrate of thickness h, as illustrated in three-dimensional form in figure 8(a) and in cross-section form in figure 8(b). Although the upper conductive strip is shown as having a finite thickness, t, in figure 8, this thickness is assumed to be negligible for the purposes of most analyses.

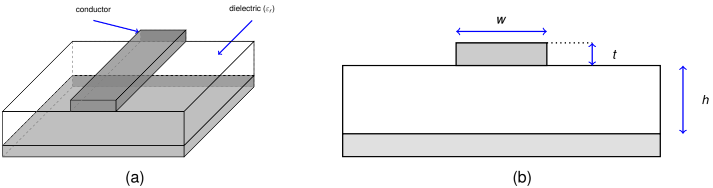

Figure 8 : Microstrip transmission line

Synthesis formulas for Microstrip

$$\frac{W}{h} = \left[\frac{e^H}{8} - \frac{1}{4e^H} \right]^{-1} \tag{10}$$

Where the parameter H is defined by :

$$H = \frac{Z_o\sqrt{2(\varepsilon_r + 1)}}{119.9} + \frac{1}{2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left[\ln\left(\frac{\pi}{2}\right) + \frac{1}{\varepsilon_r}\ln\left(\frac{4}{\pi}\right)\right] \tag{11}$$

For wide strips, i.e. $Z_o < (44 - 2\varepsilon_r)\Omega$, the synthesis formula becomes[3] :

$$\frac{W}{h} = \frac{\pi}{2}[(d_e - 1) - \ln(2d_e - 1)] + \frac{\varepsilon_r - 1}{\pi\varepsilon_r}\left[\ln(d_e - 1) + 0.293 - \frac{0.517}{\varepsilon_r}\right] \tag{12}$$

Where d_e is given by :

$$d_e = \frac{59.95\pi^2}{Z_o\sqrt{\varepsilon_r}} \tag{13}$$

Analysis formulas for Microstrip

For $W/h < 1$ we have :

$$Z_o = \frac{60}{\sqrt{\varepsilon_{eff}}} \cdot \ln\left(\frac{8h}{W} + \frac{W}{4h}\right) \tag{14}$$

where :

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\left(1 + \frac{12h}{W}\right)^{-\frac{1}{2}} + 0.04\left(1 - \frac{W}{h}\right)^2\right] \tag{15}$$

For $W/h \geq 1$ we have :

$$Z_o = \frac{\pi}{\sqrt{\varepsilon_{eff}}} \cdot \frac{120}{\frac{W}{h} + 1.393 + 0.667\ln\left(\frac{W}{h} + 1.444\right)} \tag{16}$$

where :

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{W}\right)^{-\frac{1}{2}} \tag{17}$$

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Microstrip discontinuities

- Any practical circuit implemented in microstrip must contain a number of bends, gaps and junctions if we are to connect the various components together.
- Generally speaking, such discontinuities give rise to *parasitic* capacitances and inductances which are typically quite small (often <0.1 pF and <0.1 nH). The reactance of these parasitics can, however, become significant at the high microwave and millimetre wave frequencies, so they often need to be accounted for.
- For ease of analysis, we normally model these discontinuities by an equivalent circuit consisting of lumped parasitic capacitances and inductances.

Most common types of microstrip discontinuity:

1. Open circuit 'edge' effects.
2. Series gaps.
3. Bends and curves.
4. Step width changes.

An understanding of discontinuities can actually be used deliberately in the design process to achieve a specific result. For example, a gap in a microstrip line can be used as a DC block, in place of a discrete capacitor.

Edge effects in microstrip

Silvester and Benedek[7] have modelled the open end by a small length extension, Δl, which is defined by :

$$\frac{\Delta l}{h} = 0.412 \frac{\varepsilon_{eff} + 0.3}{\varepsilon_{eff} - 0.258} = \frac{W/h + 0.262}{W/h + 0.813} \tag{18}$$

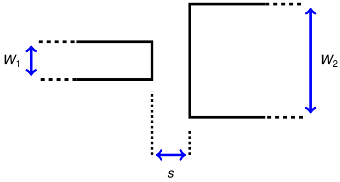
Equation (18) should suffice for most practical purposes where the edge effect needs to be taken into account, but in case greater accuracy is required, Hammerstad[4] offers an alternative equation to which he claims to be more accurate for $W/h < 20$ as follows:

$$\frac{\Delta l}{h} = 0.102 \frac{W/h + 0.106}{W/h + 0.264} \left[1.166 + \frac{\varepsilon_r + 1}{\varepsilon_r} \left(0.9 + \ln \left(\frac{W}{h} + 2.475 \right) \right) \right] \tag{19}$$

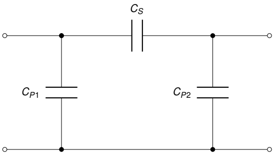
Hammerstad claims that the numerical error in equation (19) is less than 1.7% for $W/h < 20$.

Microstrip gaps and DC blocks

- It is often necessary to provide a DC break in a microstrip line, whilst allowing the RF signal to pass through with the minimum amount of attenuation. Such DC blocks are typically required between the various stages of a multi-stage amplifier, in order to isolate the DC bias levels of one stage from the next.
- The DC blocking function is often carried out using a chip capacitor connected across a gap in the microstrip line. It is possible, however, to use the gap itself as a DC block, which eliminates the need for an additional component.
- A representation of a microstrip gap between two microstrip lines of different widths is shown in figure 9(a). This gap may be approximately modelled by the equivalent π-network of figure 9(b).



(a)



(b)

Figure 9 : Microstrip gap and it's equivalent circuit

Microstrip bend

- Any practical microstrip circuit will contain a number of bends or curves, and these will also introduce parasitic effects. A simple 90^o microstrip bend is shown in figure 10(a). Such a bend can be modelled by the equivalent circuit of figure 10(b).

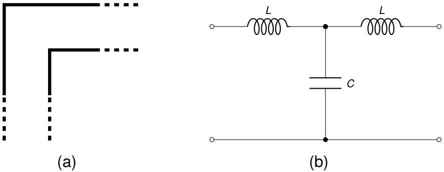


Figure 10 : Microstrip right-angle bend and its electrical equivalent circuit

Microstrip bend

According to Kirschning[5], the values of the equivalent circuit components in figure 10(b) are as follows:

$$C = W \cdot \left((10.35 \cdot \varepsilon_r + 2.5) \cdot \frac{W}{h} + (2.6 \cdot \varepsilon_r + 5.64) \right) pF \tag{20}$$

$$L = 220 \cdot h \cdot \left(1 - 1.35 \cdot \exp\left(-0.18 \cdot \left(\frac{W}{h} \right)^{1.39} \right) \right) nH \tag{21}$$

Microstrip mitred bend

- The parasitic effects of the simple 90^o bend in figure 10(a), especially the parasitic capacitor, C, in figure 10(b), can be reduced by *mitring*.
- The resultant shape is shown in figure 11(a). For a 50% mitered bend, the values of L and C are as follows[5].

$$C = W \cdot \left((3.93 \cdot \varepsilon_r + 0.62) \cdot \frac{W}{h} + (7.6 \cdot \varepsilon_r + 3.80) \right) pF \tag{22}$$

$$L = 440 \cdot h \cdot \left(1 - 1.062 \cdot \exp \left(-0.177 \cdot \left(\frac{W}{h} \right)^{0.947} \right) \right) nH \tag{23}$$

With W being width of the microstrip line and h thickness of the substrate. These formulas are valid for $W/h = 0.2$ to 6.0 and for $\varepsilon_r = 2.36$ to 10.4 and up to 14 GHz. The precision is approximately 0.3%.

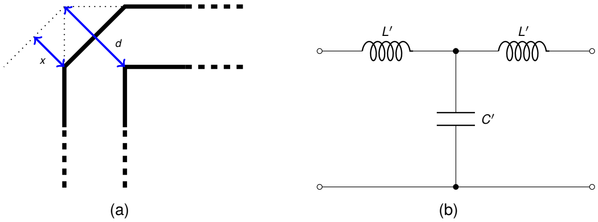


Figure 11 : Microstrip mitred bend and its electrical equivalent circuit

Microstrip mitred bend

The optimum mitre for a wide range of microstrip geometries has been determined experimentally by Douville and James[2], who show that, subject to $W/h \geq 0.25$ and $\varepsilon_r \leq 25$, a good fit for the optimum percentage mitre is given by :

$$M = 100\frac{x}{d}\% = \left(52 + 65e^{-\frac{27}{20}\frac{w}{h}}\right) \tag{24}$$

Where w is the width of the line and x and d are defined in figure 11.

- For the purposes of incorprating these bends into our circuit design, the two-port Z-matrices for the equivalent circuits in figure 10(b) and figure 11(b) can be calculated as per equation (25), where L and C represent the respective series inductances and shunt capacitance in either figure.
- The resultant Z-matrix can be converted to S-parameters using well known relationships.

$$Z = \begin{bmatrix} j\omega L + \frac{1}{j\omega C} & \frac{1}{j\omega C} \\ \frac{1}{j\omega C} & j\omega L + \frac{1}{j\omega C} \end{bmatrix} \tag{25}$$

Microstrip curve

An alternative to using a 'bend' to change the direction of a microstrip line is to use a 'curve'[8], as shown in figure 12.

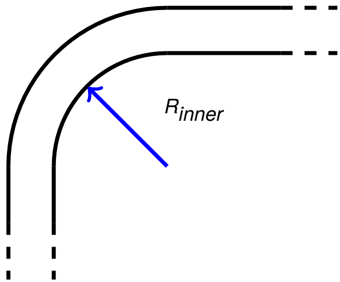


Figure 12 : 90^o curved bend in microstrip

When the curving radius is larger than twice the width of the line, the main parasitic effect is a change in the effective line length. The effective length of the curve (3 < R/W < 7) can be estimated by assuming the effective radius to be:

$$R_{eff} = R_{inner} + 0.3W \tag{26}$$

For both the curved and mitered bends, the electrical length is somewhat shorter than the physical path length of the microstrip line.

Microstrip step width change

The method of compensating for excess capacitance in a step width change is similar to that used to compensate for that in an open ended line, and is based on an expression for the length correction, l_S, required for the lower impedance line, W_2, proposed by Edwards[3] as follows:

$$\frac{l_S}{h} = 0.412\frac{\varepsilon_{eff} + 0.3}{\varepsilon_{eff} - 0.258} = \frac{W/h + 0.262}{W/h + 0.813}\left[1 - \frac{W_1}{W_2}\right] \tag{27}$$

Applying (18), this can be written as :

$$\frac{l_S}{h} = \frac{\Delta l}{h}\left[1 - \frac{W_1}{W_2}\right] \tag{28}$$

Where $\Delta l/h$ is the value of length correction required in an open ended line, obtained from (18). If more accuracy is required then (19) may be used to calculate the $\Delta l/h$ term in (28).

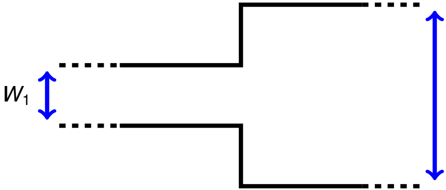


Figure 13 : Microstrip step discontinuity and electrical equivalent circuit

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Stripline

- Stripline is a form of printed circuit transmission line where the signal trace is sandwiched between an upper and lower ground plane, as illustrated in three-dimensional form in figure 14(a) and in cross-section form in figure 14(b).
- There are a number of advantages to such an arrangement, most important of which being that the electromagnetic radiation is entirely enclosed within a homogeneous dielectric, thus minimising emissions and providing natural shielding against incoming spurious signals.
- It is worth mentioning, also, that although the two 'ground' planes represent AC grounds, they can be at different DC potentials, making for convenient distribution of DC power.

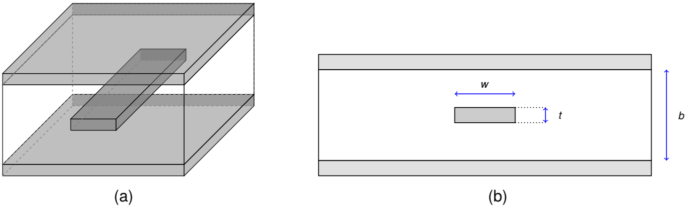

Figure 14 : Stripline transmission line

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Coplanar waveguide

- A coplanar waveguide is a printed circuit structure whereby the upper metalisation consists of a central metallic strip flanked by two narrow slits with ground plane on either side. This upper structure is separated from the ground plane by a dielectric substrate, as illustrated in three-dimensional form in figure 15(a) and in cross-section form in figure 15(b).
- The important dimensions of a coplanar waveguide are the central strip width W and the width of the slots s. The structure is normally symmetrical along a vertical plane running in the middle of the central strip (i.e. both slots are the same width).

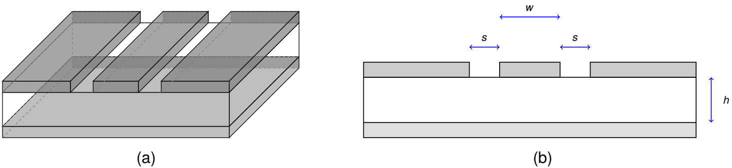

Figure 15 : Coplanar waveguide transmission line

Insertion loss : Microstrip vs coplanar waveguide

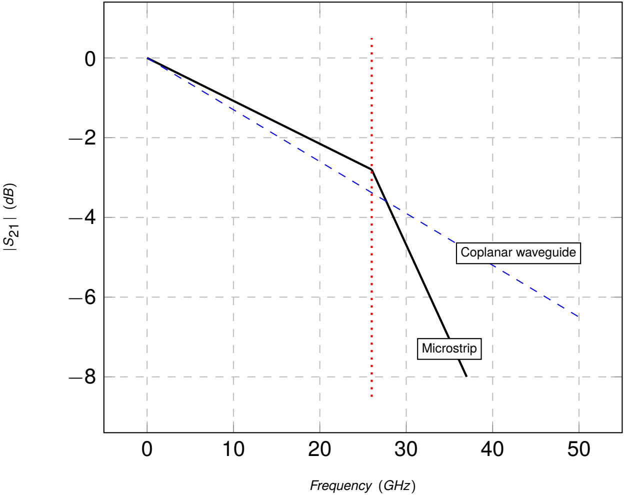

Figure 16 : Comparison of insertion loss for microstrip and coplanar waveguide

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