Lecture 5 - Immittance Parameters

Microwave Active Circuit Analysis and Design

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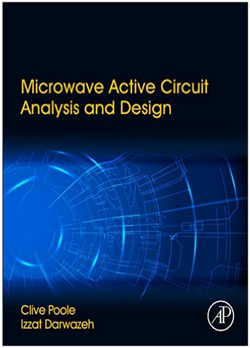

Intended Learning Outcomes

- Knowledge
  - Be familiar with the most common types of immittance parameter representations (Z, Y, h and 'ABCD' parameters), their respective strengths, weaknesses and applications.
  - Be aware of the use of Miller's theorem to simplify two-port feedback problems.
- Skills
  - Be able to calculate the input immittance of a two-port network with an arbitrary load termination, and the output immittance of a two-port network with an arbitrary source termination.
  - Be able to convert a given dataset from one immittance parameter representation to another.
  - Be able to calculate the immittance parameters of a two-port device with shunt or series feedback.
  - Be able to apply immittance parameters to determine whether a given two-port is 'active', 'passive' or 'lossless'.
  - Be able to apply the Indefinite Admittance Matrix to convert Y-parameters of one transistor configuration to another.

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The admittance or 'Y' Parameters

$$i_1 = Y_{11}v_1 + Y_{12}v_2 \tag{1}$$
$$i_2 = Y_{21}v_1 + Y_{22}v_2 \tag{2}$$

or, in matrix form :

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3}$$

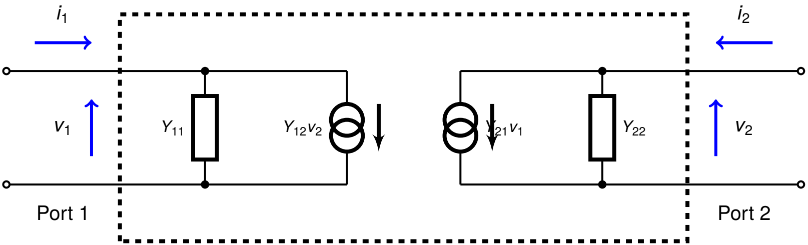

Figure 1 : Two-port Y-parameter equivalent circuit

Measurement of the 'Y' Parameters

The measurement of Y-parameters is summarised in table **??**:

Y_{11}	:	Input admittance with output short circuited
Y_{12}	:	Reverse transconductance with input short circuited
Y_{21}	:	Forward transconductance with output short circuited
Y_{22}	:	Output admittance with input short circuited

Referring to figure 1, the Y-parameters are defined as follows :

$$\left.\begin{array}{l} Y_{11} = \left.\dfrac{i_1}{v_1}\right|_{v_2=0} \\ \\ Y_{12} = \left.\dfrac{i_1}{v_2}\right|_{v_1=0} \\ \\ Y_{21} = \left.\dfrac{i_2}{v_1}\right|_{v_2=0} \\ \\ Y_{22} = \left.\dfrac{i_2}{v_2}\right|_{v_1=0} \end{array}\right\} \tag{4}$$

The impedance or 'Z' Parameters

$$v_1 = Z_{11}i_1 + Z_{12}i_2 \tag{5}$$
$$v_2 = Z_{21}i_1 + Z_{22}i_2 \tag{6}$$

or, in matrix form :

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{7}$$

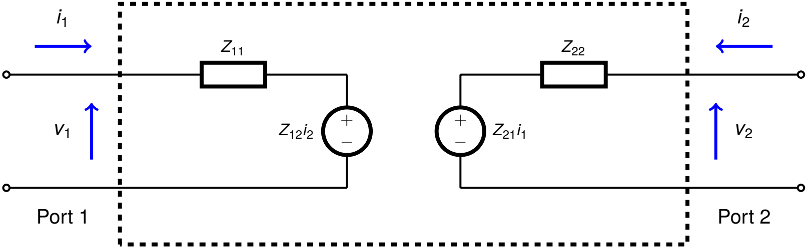

Figure 2 : Two-port Z-parameter equivalent circuit

Measurement of the 'Z' Parameters

The measurement of Z-parameters are summarised in table **??** :

Z_{11}	:	Input impedance with output open circuited
Z_{12}	:	Reverse transimpedance with input open circuited
Z_{21}	:	Forward transimpedance with output open circuited
Z_{22}	:	Output impedance with input open circuited

Refering to figure 2, the Z-parameters are defined as follows :

$$\left.\begin{aligned} Z_{11} &= \left.\frac{v_1}{i_1}\right|_{i_2=0} \\ Z_{12} &= \left.\frac{v_1}{i_2}\right|_{i_1=0} \\ Z_{21} &= \left.\frac{v_2}{i_1}\right|_{i_2=0} \\ Z_{22} &= \left.\frac{v_2}{i_2}\right|_{i_1=0} \end{aligned}\right\} \tag{8}$$

The hybrid or 'h' Parameters

$$v_1 = i_1 h_{11} + v_2 h_{12} \tag{9}$$

$$i_2 = i_1 h_{21} + v_2 h_{22} \tag{10}$$

Or in matrix form :

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \tag{11}$$

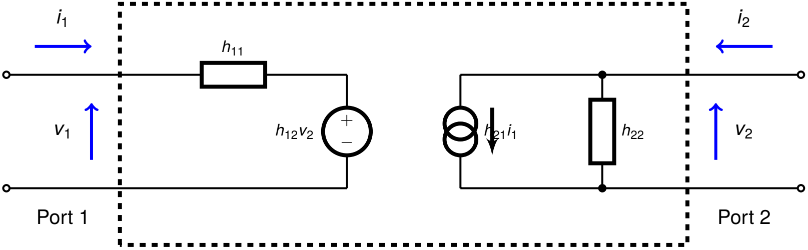

Figure 3 : Two-port *h*-parameter equivalent circuit

Measurement of the 'h' Parameters

- The h-parameters are ideally suited to the characterisation of bipolar transistors, which, in the common emitter configuration, have a low input impedance (base-emitter junction) and high output impedance (collector-emitter junction).
- For this reason, the h-parameters are widely used for transistor circuit design at lower frequencies.

h-parameters are measured as follows :

h_{11}	:	Input impedance with output short circuited
h_{12}	:	Reverse voltage transfer ratio with input open circuited
h_{21}	:	Forward current transfer ratio with output short circuited
h_{22}	:	Output admittance with input open circuited

The chain or 'ABCD' parameters

The so called chain or 'ABCD' parameters are defined by taking v_1 and i_1 as the independent variables and v_2 and i_2 as the dependent variables. The network equations in this case are:

$$v_1 = Av_2 - Bi_2 \quad (12)$$
$$i_1 = Cv_2 - Di_2 \quad (13)$$

or, in matrix form :

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \quad (14)$$

The primary application of the ABCD matrix is the calculation of the overall immittance parameters for a cascade of two or more two-port networks as shown in figure 4 on the next slide.

Cascading two-ports using ABCD-parameters

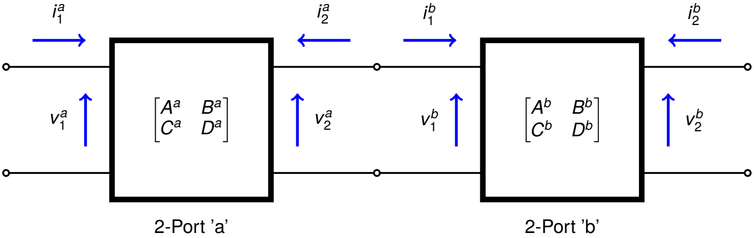

Figure 4 : Cascaded two-ports represented by ABCD-parameters

$$\begin{bmatrix} v_1^a \\ i_1^a \end{bmatrix} = \begin{bmatrix} A^a & B^a \\ C^a & D^a \end{bmatrix} \begin{bmatrix} v_2^a \\ -i_2^a \end{bmatrix} \quad (15)$$

$$\begin{bmatrix} v_1^b \\ i_1^b \end{bmatrix} = \begin{bmatrix} A^b & B^b \\ C^b & D^b \end{bmatrix} \begin{bmatrix} v_2^b \\ -i_2^b \end{bmatrix} \quad (16)$$

Cascading two-ports using ABCD-parameters

From figure 4 we can see that:

$$\begin{bmatrix} v_2^a \\ i_2^a \end{bmatrix} = \begin{bmatrix} v_1^b \\ -i_1^b \end{bmatrix} \tag{17}$$

Therefore, we can combine equations (15) and (16) to obtain:

$$\begin{bmatrix} v_1^a \\ i_1^a \end{bmatrix} = \begin{bmatrix} A^a & B^a \\ C^a & D^a \end{bmatrix} \begin{bmatrix} A^b & B^b \\ C^b & D^b \end{bmatrix} \begin{bmatrix} v_2^b \\ -i_2^b \end{bmatrix} \tag{18}$$

Or:

$$\begin{bmatrix} v_1^a \\ i_1^a \end{bmatrix} = \begin{bmatrix} A^{ab} & B^{ab} \\ C^{ab} & D^{ab} \end{bmatrix} \begin{bmatrix} v_2^b \\ -i_2^b \end{bmatrix} \tag{19}$$

Where:

$$\begin{aligned} \begin{bmatrix} A^{ab} & B^{ab} \\ C^{ab} & D^{ab} \end{bmatrix} &= \begin{bmatrix} A^a & B^a \\ C^a & D^a \end{bmatrix} \begin{bmatrix} A^b & B^b \\ C^b & D^b \end{bmatrix} \\ &= \begin{bmatrix} (A^aA^b + B^aC^b) & (A^aB^b + B^aD^b) \\ (C^aA^b + D^aC^b) & (C^aB^b + D^aD^b) \end{bmatrix} \end{aligned} \tag{20}$$

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Conversion between immittance parameters

Taking the Y and Z parameters as an example, given (3) and (7) we can write :

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{21}$$

Since the current vectors are equal, we can write :

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{22}$$

Multiplying the matrices gives us the following relationships between the various elements :

$$\left.\begin{aligned} Y_{11}Z_{11} + Y_{12}Z_{21} &= 1 \\ Y_{11}Z_{12} + Y_{12}Z_{22} &= 0 \\ Y_{21}Z_{11} + Y_{22}Z_{21} &= 0 \\ Y_{21}Z_{12} + Y_{22}Z_{22} &= 1 \end{aligned}\right\} \tag{23}$$

Conversion between immittance parameters

We can solve the simultaneous equations (23) to obtain the Y parameters in terms of Z parameters or vice versa, as follows :

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \left(\frac{Z_{22}}{Z_{11}Z_{22}-Z_{12}Z_{21}}\right) & \left(\frac{-Z_{12}}{Z_{11}Z_{22}-Z_{12}Z_{21}}\right) \\ \left(\frac{-Z_{21}}{Z_{11}Z_{22}-Z_{12}Z_{21}}\right) & \left(\frac{Z_{11}}{Z_{11}Z_{22}-Z_{12}Z_{21}}\right) \end{bmatrix} \tag{24}$$

and

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \left(\frac{Y_{22}}{Y_{11}Y_{22}-Y_{12}Y_{21}}\right) & \left(\frac{-Y_{12}}{Y_{11}Y_{22}-Y_{12}Y_{21}}\right) \\ \left(\frac{-Y_{21}}{Y_{11}Y_{22}-Y_{12}Y_{21}}\right) & \left(\frac{Y_{11}}{Y_{11}Y_{22}-Y_{12}Y_{21}}\right) \end{bmatrix} \tag{25}$$

We can carry out similar analysis for the h and ABCD parameters.

Input and output impedance of a two-port

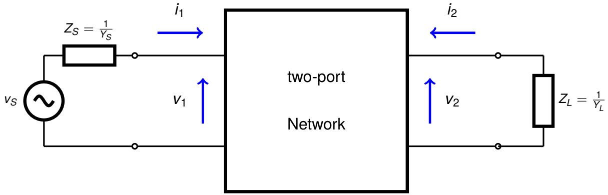

Figure 5 : Two-port network with load and source terminations

Examining figure 5 we can write :

$$-i_2 = Y_L v_2 \tag{26}$$

Combining (1) and (26) we have :

$$\frac{v_2}{v_1} = \frac{-Y_{21}}{Y_{22} + Y_L} \tag{27}$$

Substituting for v_2 in (1) we have :

$$i_1 = Y_{11} v_1 - \frac{Y_{21} Y_{12}}{Y_{22} + Y_L} v_1 \tag{28}$$

Hence we can write the input admittance, Y_{in}, as :

$$Y_{in} = \frac{i_1}{v_1} = Y_{11} - \frac{Y_{21} Y_{12}}{Y_{22} + Y_L} \tag{29}$$

Input and output impedance of a two-port

The reader may like to undertake a similar analysis to prove that the output admittance, Y_{out}, is given by :

$$Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11} + Y_S} \tag{30}$$

(Note that the voltage source v_S can be treated as a short circuit for the purpose of determining Y_{out}.)
If we prefer to work with Z-parameters, we could follow a similar line of analysis to the above which would result in the following :

$$Z_{in} = Z_{11} - \frac{Z_{21}Z_{12}}{Z_{22} + Z_L} \tag{31}$$

$$Z_{out} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11} + Z_S} \tag{32}$$

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Activity and Passivity

Consider the two-port of figure 6 on page 19. The total power entering the two-port is given by :

$$P_{tot} = Re(V_1 I_1^*) + Re(V_2 I_2^*) \quad (33)$$
$$= Re(V_1^* I_1) + Re(V_2^* I_2) \quad (34)$$

Where the asterisk superscript denotes conjugate and $Re(x)$ denotes the real part of the complex quantity, x.
(33) can be rewritten as :

$$P_{tot} = \frac{1}{2}(V_1 I_1^* + V_1^* I_1) + \frac{1}{2}(V_2 I_2^* + V_2^* I_2) \quad (35)$$

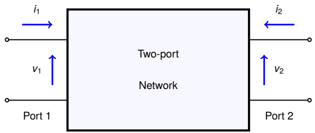

Figure 6 : Two-port network with port definitions

Activity and Passivity

Equation (35) can be rearranged to give :

$$P_{tot} = \frac{1}{2}(V_1^* I_1 + V_2^* I_2) + \frac{1}{2}(I_1^* V_1 + I_2^* V_2) \quad (36)$$

which can be written in matrix form as :

$$P_{tot} = \frac{1}{2}[V^*]^T[I] + \frac{1}{2}[I^*]^T[V] \quad (37)$$

Where $[]^T$ denotes the conjugate transpose of a matrix. We now introduce the Y-parameters by employing the following relationships :

$$[I] = [Y][V]$$
$$[I^*]^T = [V^*]^T[Y^*]^T$$

We can now write (37) as follows :

$$P_{tot} = \frac{1}{2}[V^*]^T[Y][V] + \frac{1}{2}[V^*]^T[Y^*]^T[V] \quad (38)$$

Which can be factored to give :

$$P_{tot} = [V^*]^T\left[\frac{1}{2}([Y] + [Y^*]^T)\right][V] \quad (39)$$

Activity and Passivity

- The term $1/2([Y] + [Y^*]^T)$ is called a *Hermitian form* of the admittance matrix (in this case).
- A Hermitian matrix is a square matrix with complex entries that is equal to its own conjugate transpose. This means that the elements along the principal diagonal are real, and the complex elements on each side of the principal diagonal are complex conjugates.
- Passivity requires $P_{tot} \geq 0$ for any $V_i \neq 0$. Which requires that the Hermitian form in (39) must be *positive semidefinite*. This means that all principal minors in the Y-parameter determinant, which is the set of subdeterminants that can be taken symmetrically around the diagonal, are positive or zero.

$$Det\left[\frac{1}{2}([Y] + [Y^*]^T)\right] = \begin{vmatrix} G_{11} & \frac{1}{2}(Y_{12} + Y_{21}^*) \\ \frac{1}{2}(Y_{21} + Y_{12}^*) & G_{22} \end{vmatrix} \tag{40}$$

Where : $Y_{ij} = G_{ij} + jB_{ij}$

Activity and Passivity

Expanding the determinant of (40) gives us three conditions for passivity, as follows :

$$G_{11} \geq 0 \quad (41)$$

$$G_{22} \geq 0 \quad (42)$$

$$G_{11}G_{22} - \frac{1}{4}(Y_{21} + Y_{12}^{*})(Y_{12} + Y_{21}^{*}) \geq 0 \quad (43)$$

Condition 43 can be rewritten as :

$$4(G_{11}G_{22} - G_{12}G_{21}) - |Y_{21} - Y_{12}|^2 \geq 0 \quad (44)$$

From 44 we can write the passivity condition as :

$$0 \leq U \leq 1 \quad (45)$$

Where U is defined as :

$$U = \frac{|Y_{21} - Y_{12}|^2}{4(G_{11}G_{22} - G_{12}G_{21})} \quad (46)$$

The quantity U is referred to as *Mason's unilateral power gain* or *Mason's invariant*, and, as the name suggests, has a wider significance than just as an indication of passivity.

Activity and Passivity

Equation 46 can be more succinctly written in matrix form as follows :

$$U = \frac{\left|\det\left[Y - Y^T\right]\right|}{\det\left[Y + Y*\right]} \tag{47}$$

or in Z-parameter form as :

$$U = \frac{\left|\det\left[Z - Z^T\right]\right|}{\det\left[Z + Z*\right]} \tag{48}$$

which equates to :

$$U = \frac{\left|Z_{21} - Z_{12}\right|^2}{4(R_{11}R_{22} - R_{12}R_{21})} \tag{49}$$

Where : $Z_{ij} = R_{ij} + jX_{ij}$
Mason's invariant 'U' as defined by equations (46) to (49) is the only device characteristic that is invariant under lossless, reciprocal 'embeddings', meaning that the active device is embedded within some other lossless, reciprocal network (such as a passive feedback network). This means that U can be used as a figure of merit to compare any three-terminal, active device.
U can also be used as a criterion for activity/passivity : if $U > 1$, then the two-port device is active; otherwise, the device is passive.

Activity and Passivity

A special case of the passive network is the *lossless* network, where the total power entering the two-port is zero, i.e. :

$$P_{tot} = 0 \tag{50}$$

It can be shown that the condition for losslessness is that the network contains no resistive elements. The inequalities in conditions (41) to (43) are then replaced by an equality, i.e. :

$$G_{11} = 0 \tag{51}$$

$$G_{22} = 0 \tag{52}$$

$$G_{11}G_{22} - \frac{1}{4}(Y_{21} + Y_{12}^*)(Y_{12} + Y_{21}^*) = 0 \tag{53}$$

Which leads to the condition for losslessness in terms of Mason's invariant 'U', as :

$$U = 1 \tag{54}$$

Unilaterality

At this point it, is worth pausing to consider the implications of equations (29) through (32). If the reverse transfer parameter, Y_{12} or Z_{12}, is zero then the second term in (29) to (32) disappears. The input and output immittances then simply reduce to :

$$Y_{in} = Y_{11} \quad (55)$$
$$Y_{out} = Y_{22} \quad (56)$$

and :

$$Z_{in} = Z_{11} \quad (57)$$
$$Z_{out} = Z_{22} \quad (58)$$

In the case of a zero reverse transfer parameter, the input impedance is effectively de-coupled from the load, and the output impedance is effectively de-coupled from the source. In this case, the device is referred to as being *unilateral*.

Reciprocity and Symmetry

A two-port is said to be *reciprocal* when the reverse and forward transfer parameters are identical, i.e. :

Table 1 : Reciprocity conditions

Z-parameters	:	$Z_{12} = Z_{21}$
Y-parameters	:	$Y_{12} = Y_{21}$
h-parameters	:	$h_{12} = h_{21}$
ABCD-parameters	:	$AD - BC = 1$

A special case of reciprocity is *symmetry*, where the two-port behaves identically when ports 1 and 2 are interchanged. The conditions for symmetry, in addition to the reciprocity conditions in table 1 above, are as follows :

Table 2 : Symmetry conditions

Z-parameters	:	$Z_{11} = Z_{22}$
Y-parameters	:	$Y_{11} = Y_{22}$
h-parameters	:	$\Delta h = h_{11}h_{22} - h_{21}h_{12} = 1$
ABCD-parameters	:	$A = D$

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Two-port representation of transistors

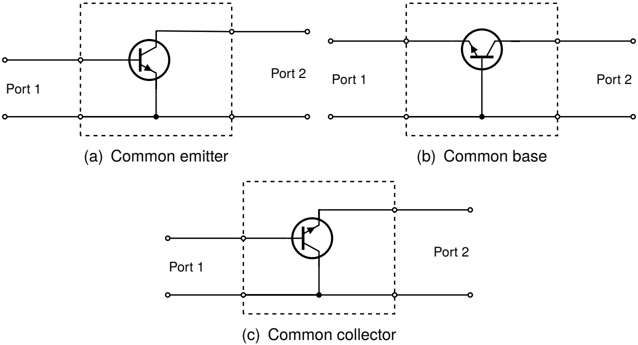

Figure 7 : Transistor two-port configurations

Two-port representation of transistors

- The convention is, when dealing with transistors, to give the parameters the subscript '*ij*', where i indicates the terminal or direction of the measurement and j denotes the transistor configuration.
- The direction subscript, *i*, can take one of four possible values, namely : *i*=input, *f*=forward, *r*=reverse, *o*=output.
- For example, the *h*-parameters for the three possible transistor configurations are renamed as in table 3:

Table 3 : *h*-parameter transistor subscripts

h-parameter	Common Emitter	Common Base	Common Collector
h_{11}	h_{ie}	h_{ib}	h_{ic}
h_{12}	h_{re}	h_{rb}	h_{rc}
h_{21}	h_{fe}	h_{fb}	h_{fc}
h_{22}	h_{oe}	h_{ob}	h_{oc}

Two-port representation of transistors

- It is sometimes advantageous to be able to convert the two-port h-parameters for one configuration into two-port parameters for the other two.
- Assuming we started with the common emitter *h*-parameters for a given transistor, the *h*-parameters for the same transistor in common base and common collector configurations would be as follows :

Table 4 : Transistor *h*-parameter conversion formulae

h-parameter	Common Emitter	Common Base	Common Collector
h_{11}	h_{ie}	$h_{ib} = \frac{h_{ie}}{1 + h_{fe}}$	$h_{ic} = h_{ie}$
h_{12}	h_{re}	$h_{rb} = \frac{h_{ie}h_{oe}}{1 + h_{fe}} - h_{re}$	$h_{rc} = 1 - h_{re}$
h_{21}	h_{fe}	$h_{fb} = \frac{-h_{fe}}{1 + h_{fe}}$	$h_{fc} = -(1 + h_{fe})$
h_{22}	h_{oe}	$h_{ob} = \frac{h_{oe}}{1 + h_{fe}}$	$h_{oc} = h_{oe}$

Transistor h-parameter example

As an indication, the typical *h*-parameters for a small-signal Bipolar Junction Transistor at low frequencies are as follows :

$$h_{ie} = 1k\Omega$$
$$h_{re} = 3 \times 10^{-4}$$
$$h_{fe} = 250$$
$$h_{oe} = 3 \times 10^{-6} siemens$$

Using the formulae in table 4, the above device will have the following parameters :

Table 5 : Example transistor *h*-parameters

h-parameter	Common Emitter	Common Base	Common Collector
h_{11}	$1k\Omega$	3.98Ω	$1k\Omega$
h_{12}	3×10^{-4}	-2.88×10^{-4}	1
h_{21}	250	-0.996	-251
h_{22}	$3 \times 10^{-6} siemens$	$1.20 \times 10^{-8} siemens$	$3 \times 10^{-6} siemens$

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Two-ports with feedback

The application of feedback to an active two-port effectively adds another degree of freedom to the design process. If the two-port parameters of the transistor in question are not to your liking, you can change them by applying feedback and then proceed with your design using a new set of two-port parameters describing the transistor plus feedback.

In general, feedback is applied to achieve one or more of the following objectives :

1. Increase stability.
2. Extend the bandwidth by 'flattening' the frequency response.
3. Alter the input and/or output impedance to improve matching.
4. To generate negative resistance at one of the ports (in the case of oscillator design).

The above improvements often come at a cost, which is usually in the form of a reduction in power gain.

The study of feedback is a significant branch of electronic engineering in its own right, and is extensively covered elsewhere. We will only provide here a brief description of the two main types of two-port feedback used in RF circuits, namely shunt feedback and series feedback.

Shunt feedback

$$[Y] = [Y] + [Y_{fb}] = \begin{bmatrix} Y_{11} + Y_{11}^{fb} & Y_{12} + Y_{12}^{fb} \\ Y_{21} + Y_{21}^{fb} & Y_{22} + Y_{22}^{fb} \end{bmatrix} \quad (59)$$

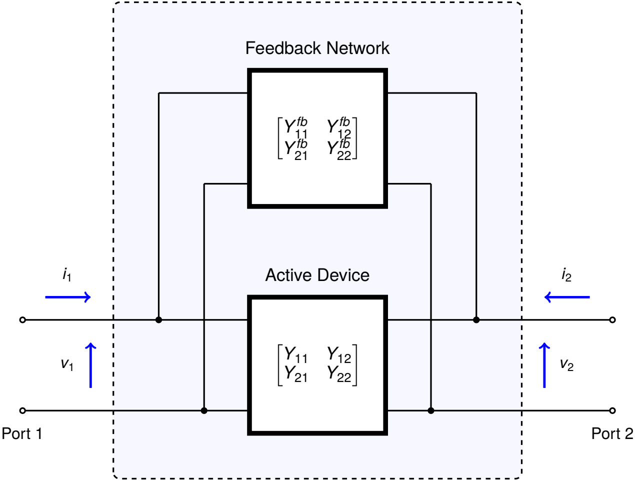


Figure 8 : Two-port network, $[Y]$, with shunt feedback network, $[Y^{fb}]$, added

Series feedback

$$[Z] = [Z] + [Z_{fb}] = \begin{bmatrix} Z_{11} + Z_{11}^{fb} & Z_{12} + Z_{12}^{fb} \\ Z_{21} + Z_{21}^{fb} & Z_{22} + Z_{22}^{fb} \end{bmatrix} \quad (60)$$

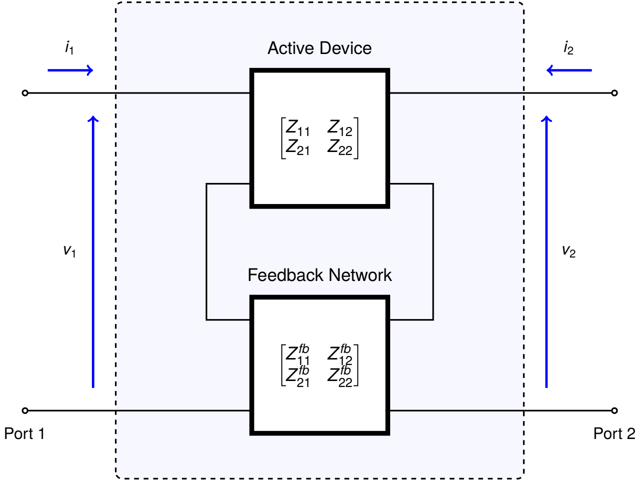


Figure 9 : Two-port network, $[Z]$, with series feedback network, $[Z^{fb}]$, added

Miller's theorem

The values of the equivalent admittances shown in figure 10(b), in terms of Y_f, are given by :

$$Y_1 = Y_f(1 - A_v) \qquad (61)$$

$$Y_2 = Y_f\left(\frac{A_v - 1}{A_v}\right) \qquad (62)$$

where the voltage gain A_v is defined as :

$$A_v = \left.\frac{v_2}{v_1}\right|_{i_2=0} \qquad (63)$$

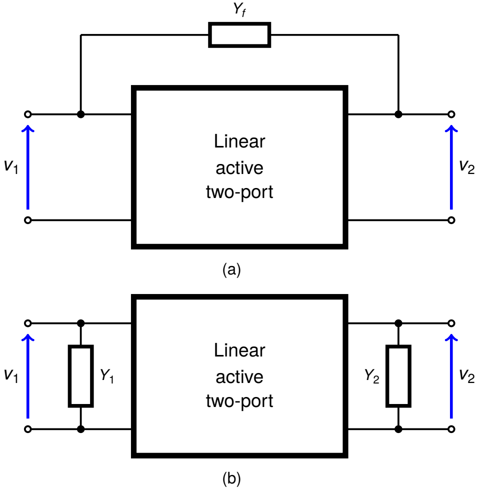


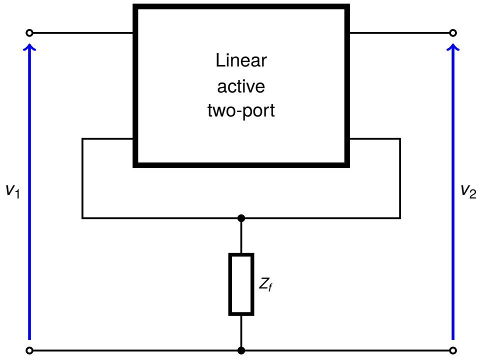
Figure 10 : Miller's theorem : (a) Immitance two-port with shunt feedback. (b) Equivalent circuit, according to Miller's theorem

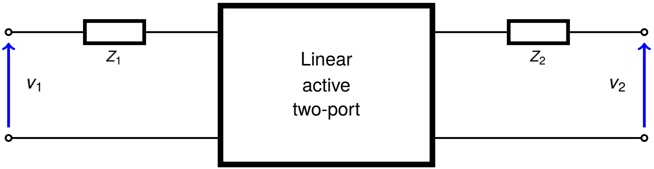
Miller's theorem (dual)

The values of the equivalent impedances Z_1 and Z_2, in terms of Z_f, are given by :

$$Z_1 = Z_f(1 - A_v) \quad (64)$$

$$Z_2 = Z_f\left(\frac{A_v - 1}{A_v}\right) \quad (65)$$





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