Lecture 9 - Lumped Element Matching Networks

Microwave Active Circuit Analysis and Design

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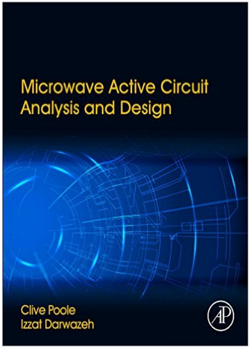

Intended Learning Outcomes

- Knowledge
  - Understand the theory of L-section matching networks both analytically and graphically.
  - Understand forbidden regions on the Smith Chart for various types of L-section.
  - Understand the theory of three element matching networks (π-section and T-section).
  - Understand bandwidth performance of lumped element matching networks.
- Skills
  - To be able to select the appropriate L-section matching network (type 1 or type 2) depending on the values of impedances to be matched.
  - Be able to design an L-section matching network to match two arbitrary impedances using either the analytical or Smith Chart (graphical) approach.
  - Be able to design a T or π matching network to match two arbitrary impedances.

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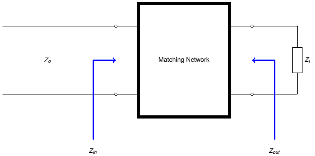
Impedance Matching

An impedance matching network is used between two dissimilar impedances in order to ensure maximum power transfer between them. We typically want to match an arbitrary load Z_L to a transmission line Z_o. The conditions for maximum power transfer are therefore :



$$\boxed{Z_{in}^{*} = Z_o}$$

and

$$\boxed{Z_{out}^{*} = Z_L}$$

The need for Impedance Matching :

- In many applications we require Maximum Power Transfer into the load. This is achieved when the load to be matched to the line.
- An impedance matching network is required to present the optimum source impedance to the input of a low noise amplifier, in order to achieve minimum noise figure.

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L-Section Matching Networks

- This is the simplest lumped component matching network, consisting of only two lumped components : a shunt susceptance, jB, and a series reactance, jX.
- There are two basic configurations of L-Section matching network, depending on the location of the shunt element. We shall refer to these as 'type 1' and 'type 2' :

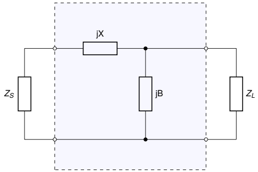


Figure 1 : L-section : Type 1

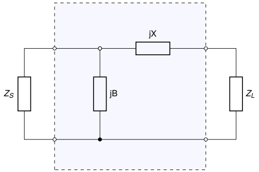
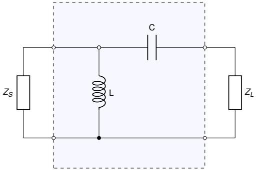


Figure 2 : L-section : Type 2

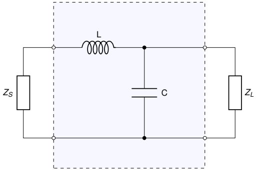
Generally speaking, the shunt element, jB, is placed in parallel with the larger of Z_S or Z_L.

High pass vs Low pass L-Section networks

- High Pass Configuration :
  - When the series component is a capacitor, it will block DC into the load.
  - The shunt inductor will act as a short at low frequencies.



- Low Pass Configuration :
  - When the series component is an inductor, it will allow DC into the load, but will attenuate higher frequencies.
  - The shunt capacitor will act as a short at high frequencies.



L-section matching of a resistive source to a resistive load

Let us assume that we need to connect a resistive load R_L to a resistive source R_S, as shown in figure 3(a), where, in the general case, $R_S \neq R_L$.

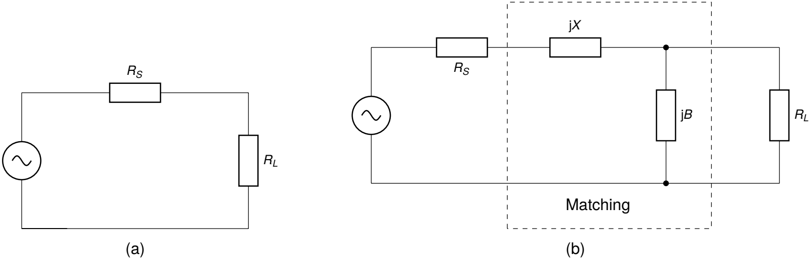


Figure 3 : Matching arbitrary R_L and R_S

L-section matching of a resistive source to a resistive load

We need to place a matching network between R_S and R_L, as in figure 3(b), to ensure that the source is terminated by an impedance equal to itself, thereby ensuring maximum power transfer.
Since we restrict ourselves to using only reactive elements in the matching network, the matching of the two resistive elements proceeds as follows :

1. Firstly, we place a reactive element, represented by the susceptance jB, in parallel with R_L, such that the resistive part of the resulting combination is equal to R_S.
2. We then cancel the reactive part of the combination ($jB \parallel R_L$) by adding the equal and opposite series reactive element jX.

L-section matching of a resistive source to a resistive load

We can analyse the circuit of figure 3(b) as follows : Since we know that, to satisfy the matching condition, the total impedance of the parallel combination ($jB \parallel R_L$) should be the complex conjugate of $R_S + jX$, we can write :

$$R_S + jX = \frac{1}{(1/R_L) - jB} = \frac{R_L + jBR_L^2}{1 + B^2 R_L^2} \tag{1}$$

From (1) we get R_S and X in terms of R_L and B as follows :

$$R_S = \frac{R_L}{1 + B^2 R_L^2} \tag{2}$$

and

$$X = \left(\frac{BR_L^2}{1 + B^2 R_L^2} \right) \tag{3}$$

L-section matching of a resistive source to a resistive load

Equation (**??**) implies that a real value of Q is only obtained if $R_L/R_S > 1$. If this is not the case then we need to reverse the position of X and B in figure 3(b), in other words B is placed in parallel with the source, not the load. We can apply exactly the same design procedure, only treating the source as if it were the load and vice versa. We can therefore write (**??**) in a form that covers any values of R_S and R_L as :

$$Q = \sqrt{\left(\frac{R_{high}}{R_{low}}\right) - 1} \tag{4}$$

Where R_{high} is the higher value of R_S and R_L and R_{low} is the lower value. Another way of intuitively understanding where the parallel arm should be placed is to consider that, if $R_L > R_S$ then R_L needs to be reduced by adding a parallel resistance. On the other hand, if $R_L < R_S$ then it needs to be increased by adding a series resistance.

L-section matching of a resistive source to a resistive load

We can now set out the basic design procedure for an L-section to match resistive loads as follows :

1. Calculate the Q for a given R_S and R_L using (**??**) (Note the orientation of the parallel arm based on whether (R_L/R_S) is greater or less than unity).
2. Calculate B from :

$$B = \pm \frac{Q}{R_L} \tag{5}$$

3. Calculate X from (**??**).

Note that the sign of B in step 2 above may be chosen arbitrarily, since the load is purely resistive and we are free to choose B to be either capacitive or inductive. The difference being that the type of reactance chosen for B will determine whether the L-section has a high or low pass frequency characteristic away from the centre frequency. If a negative value of B is chosen (i.e. parallel capacitance) then the L-section will have a low pass characteristic. If a positive value of B is chosen (i.e. parallel inductance) then the L-section will have a high pass characteristic.

L-section matching of a complex source and load

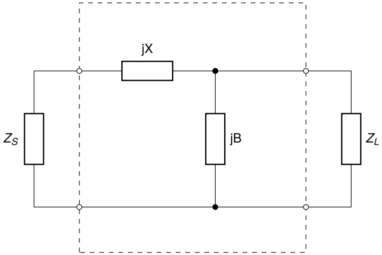
For simplicity, we will restrict our analysis to the most common situation, namely where we need to match the complex load Z_L to the system characteristic impedance, Z_o. As before, the choice of whether to use a type 1 or type 2 matching network will depend on the relationship between the resistive part of the load, R_L, and Z_o. As was shown for the case of purely resistive loads, the parallel element, jB, should be placed in parallel with whichever is larger of R_L or Z_o, in other words :

If $R_L > Z_o$: use type 1 L-section (shunt element is next to the load).

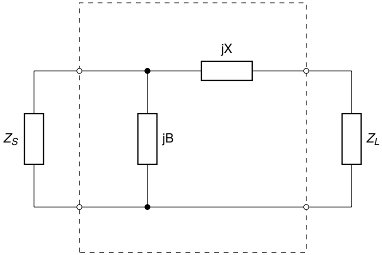
If $R_L < Z_o$: use type 2 L-section (shunt element is next to the source).

L-section matching of a complex source and load

In general, the load and source will be complex. We can generalise the above technique to cover a complex Z_L and Z_S as shown in figure 4 by considering only the resistive parts of Z_L and Z_S first and then absorbing the reactive parts into the resulting matching components X and B.



(a) L-section : Type 1

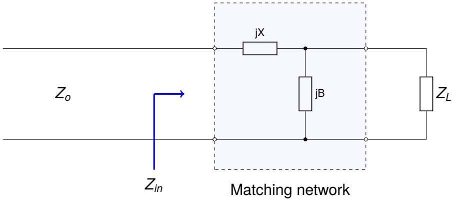


(b) L-section : Type 2

Figure 4 : Generalised L-section matching networks

Analytical design of Type 1 L-section

Consider a generalised type 1 L-section used to connect an arbitrary load, Z_L to a transmission line of characteristic impedance Z_o :



From the figure above we have three equations with 2 unknowns :

$$
\begin{aligned}
Z_L &= R_L + jX_L && (known) \\
Z_{in} &= jX + \frac{1}{jB + (R_L + jX_L)^{-1}} && (B, X : unknown) \\
Z_S &= Z_o && (known)
\end{aligned}
\quad (6)
$$

Matching is defined as $Z_{in} = Z_S$. This implies $Z_{in} = Z_o$.

Analytical design of Type 1 L-section

From the previous slide :

$$Z_o = jX + \frac{1}{jB + (R_L + jX_L)^{-1}} \tag{7}$$

The above can be rearranged and separated into real and imaginary parts to yield the following pair of equations :

$$B(XR_L + jX_L Z_o) = R_L - Z_o \tag{8}$$

$$X(1 - BX_L) = BZ_o R_L - X_L \tag{9}$$

Which can be solved to yield :

$$\boxed{B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_o}}\sqrt{R_L^2 + X_L^2 - Z_o R_L}}{R_L^2 + X_L^2}} \tag{10}$$

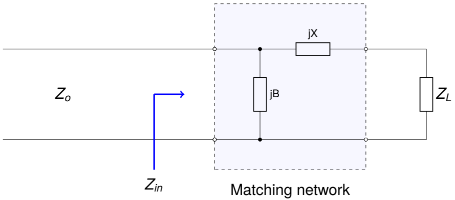
Once B is determined, X can be found using equation (9):

$$\boxed{X = \frac{BZ_o R_L - X_L}{1 - BX_L}} \tag{11}$$

Note that the term inside the square root in equation (10) can be negative. Therefore type 1 is only used in the case when $R_s > R_L$.

Analytical design of Type 2 L-section

Consider a generalised type 2 L-section used to connect an arbitrary load, Z_L to a transmission line of characteristic impedance Z_o :



From the figure above we have three equations with 2 unknowns :

$$\begin{aligned} Z_L &= R_L + jX_L && (known) \\ Y_{in} &= jB + \frac{1}{R_L + j(X + X_L)} && (B, X : unknown) \\ Z_S &= Z_o && (known) \end{aligned} \quad (12)$$

Matching is defined as $Z_{in} = Z_S$. This implies $Z_{in} = Z_o$.

Analytical design of Type 2 L-section

From the previous slide :

$$Y_{in} = jB + \frac{1}{R_L + j(X + X_L)} \tag{13}$$

The above can be rearranged and separated into real and imaginary parts to yield the following pair of equations :

$$BZ_o(X + X_L) = Z_o - R_L \tag{14}$$

$$(X + X_L) = BZ_oR_L \tag{15}$$

Which can be solved to yield :

$$\boxed{B = \pm\frac{1}{Z_o}\sqrt{\frac{(Z_o - R_L)}{R_L}}} \tag{16}$$

and

$$\boxed{X = \pm\sqrt{R_L(Z_o - R_L)} - X_L} \tag{17}$$

Once again, the requirement that $R_L < Z_o$ ensures that the terms under the square roots in the expressions for B and X and are real. Again, with two solution pairs for B and X, there are two different matching network solutions.

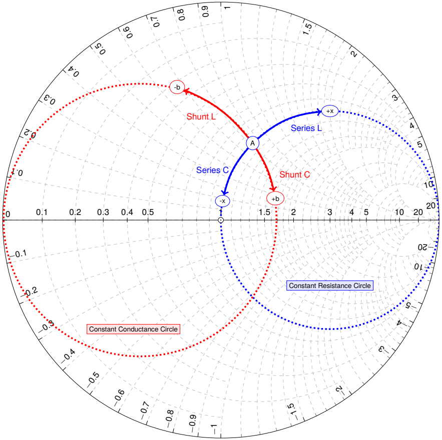
The effect of adding reactive elements

L-Section design is best performed using the Admittance/Impedance Smith chart of figure **??**.

- Adding series reactive loads will modify the impedance by adding negative reactance (series C), or positive reactance (series L)
- Adding shunt reactive loads will modify the admittance by adding negative susceptance (shunt C), or positive susceptance (shunt L)

Let's say we start at point 'A' :

- Adding a **series** component:
  - move along constant resistance circle
  - Series L : move *clockwise*
  - Series C : move *counter-clockwise*
- Adding a **shunt** component:
  - move along constant conductance circle
  - Shunt L : move *counter-clockwise*
  - Shunt C : move *clockwise*
- Generally, the point is to arrive at the origin ($Z_{in} = 50\Omega$)



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The effect of adding reactive elements

From the previous slide we can draw some general conclusions :

1. only the unit resistance and unit conductance circles pass through the origin, therefore :
   1.1 Adding series L or C alone will only match those loads lying on the unit resistance circle.
   1.2 Adding shunt L or C alone will only match those loads lying on the unit conductance circle.
2. If the load is inductive (i.e. it lies in the upper half of the Smith Chart) then we need to add series C or shunt C to match.
3. If the load is capacitive (i.e. it lies in the lower half of the Smith Chart) then we need to add series L or shunt L to match.

Points 2 and 3 above are kind of obvious : we need to add opposite reactances to cancel out the load reactance.

Unit resistance / conductance circles

There are two important circles on the Smith chart that we need to be aware of when designing L-sections :

- Unit resistance circle : the locus of all impedances of the form $z = 1 \pm jx$
- Unit conductance circle : the locus of all admittances of the form $y = 1 \pm jb$

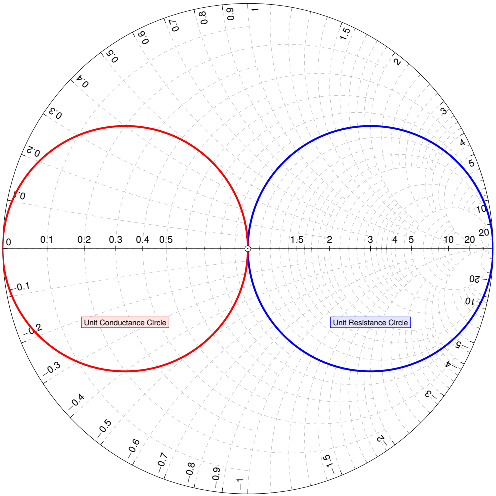
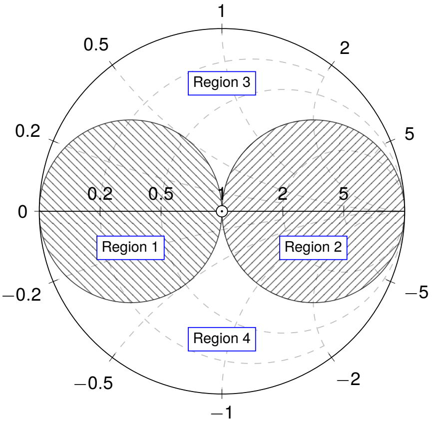


Figure 5 : Unit conductance and resistance circles

Demarcation regions on the Smith Chart

- The choice of matching network topology depends on the location of the load, z_L, on the Smith chart.
- Richard Li [?] has proposed the division of the Smith Chart into 4 distinct regions as follows :

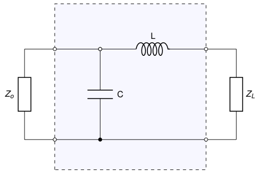
  Region 1: Low resistance or High conductance loads
  Region 2: High resistance or Low conductance loads
  Region 3: Low resistance and Low conductance loads
  Region 4: Low resistance and Low conductance loads

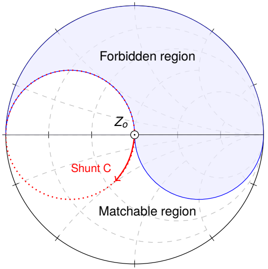


Region 1	Region 2	Region 3	Region 4
r<1 x<|0.5| g>1 $-\infty < b < +\infty$	r>1 $-\infty < x < +\infty$ g<1 b<|0.5|	r<1 x>0 g<1 b<0	r<1 x<0 g<1 b>0

Forbidden Regions for L-Sections

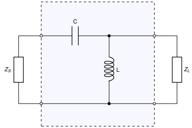
- Not every L-network topology can perform the required matching between arbitrary load and source impedances.
- There is no guarantee that a solution exists for any given L-section. Depending on the L-Section topology, there will be Forbidden Areas on the Smith Chart which cannot be matched.
- For example, assuming a 50Ω source, addition of a shunt capacitance will result in moving clockwise away from the origin along the constant conductance circle , as shown.
- This implies that all load impedances within the shaded region opposite cannot be matched with this particular configuration.



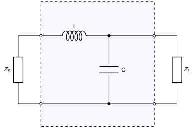


Forbidden Region: L-section type 1 (LC)

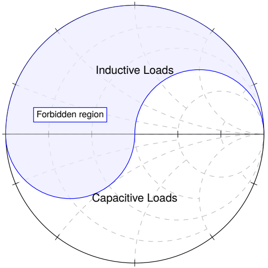
Type 1 L-Section has the series element next to the source and the parallel element next to the load.

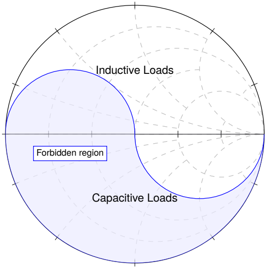


Type 1a can only match capacitive loads lying outside unit admittance circle or inductive loads lying inside unit resistance circle



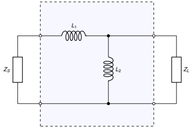
Type 1b can only match inductive loads lying outside unit admittance circle or capacitive loads lying inside unit resistance circle



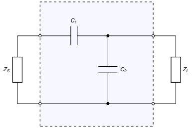


Forbidden Region: L-section type 1 (LL/CC)

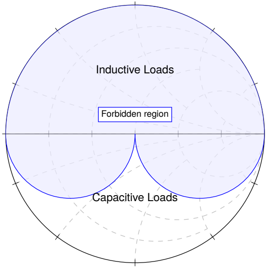
Type 1 L-Section has the series element next to the source and the parallel element next to the load.

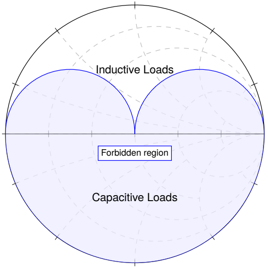


Type 1c can only match capacitive loads lying outside both unit circles



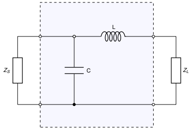
Type 1d can only match inductive loads lying outside both unit circles



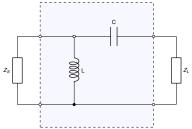


Forbidden Regions : L-section type 2 (LC)

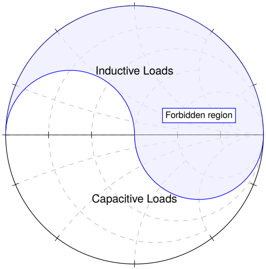
Type 2 L-Section has the series element next to the load and the parallel element next to the source.

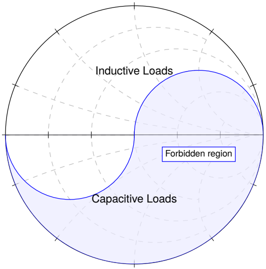


Type 2a can only match capacitive loads lying outside unit resistance circle or inductive loads lying inside unit admittance circle



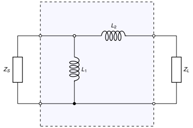
Type 2b can only match inductive loads lying outside unit resistance circle or capacitive loads lying inside unit admittance circle



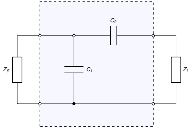


Forbidden Region: L-section type 2 (LL/CC)

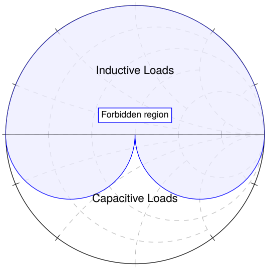
Type 1 L-Section has the series element next to the source and the parallel element next to the load.

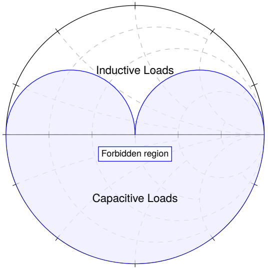


Type 2c can only match capacitive loads lying outside both unit circles



Type 2d can only match inductive loads lying outside both unit circles





4-Step Design procedure for L-Sections

Step 1: Normalize ($z_L = Z_L/50$) and locate z_L on the Smith Chart

Step 2: Select the appropriate L-section topology, based on where Z_L lies in relation to the various forbidden regions.

For type 1 L-section

Step 3: Move along the constant conductance circle until it intersects with the unit resistance circle. Record the susceptance change and thus determine the value of shunt L or C.

Step 4: Move along the unit resistance circle to the origin, record the reactance change and thus determine the value of series L or C.

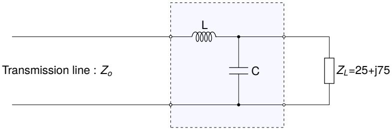
For type 2 L-section

Step 3: Move along the constant resistance circle until it intersects with the unit conductance circle. Record the reactance change and thus determine the value of series L or C.

Step 4: Move along the unit conductance circle to the origin, record the susceptance change and thus determine the value of shunt L or C.

L-Section design example

Match a load Z_L=(25 + j75)Ω to 50Ω at 10GHz.



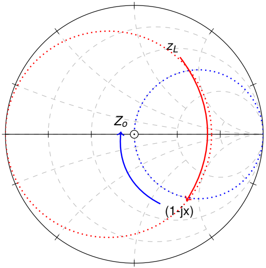
Step 1: Normalize z_L = (25 + j75)/50 = 0.5 + j1.5 and locate this point on the Smith Chart

Step 2: Based on the location of z_L we select a type 1 LC L-section.

Step 3: Move clockwise along the constant conductance circle until it intersect the unit resistance circle at 1-jx. Susceptance change $= \frac{1}{-100} - \frac{1}{75} = -0.023℧$.

Therefore $C = \frac{0.023}{2\pi f} = \boxed{0.371pF}$

Step 4: Move clockwise along the unit resistance circle to the origin. Reactance change =100Ω. Therefore $L = \frac{100}{2\pi f} = \boxed{1.59nH}$.



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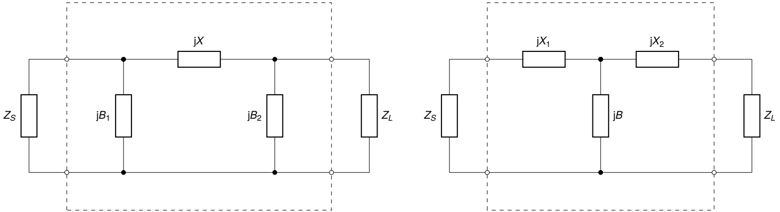
3 element matching networks

- L-sections provide no flexibility over the circuit Q, which is a function of the load and source impedances.
- High ratios of source and load impedance will have high Q, resulting in a narrower bandwidth than we would like.
- In order to have control over circuit Q we need to add more flexibility by going from 2-elements to 3-elements in the matching network.
- There are basically two types of 3-element matching network defined by the topology : the ’T-network’ and the ’π-network’.

Three element matching network

We sometimes need to specify a Q value for the matching network, in which case, we need more degrees of freedom in the design and this requires more circuit elements. The next step up in complexity from the two element L-sections just described is the three element matching network.

There are two basic configurations of three element matching network, which are referred to as the π-section and the T-section, according to their respective topologies, as shown in figure 6.



(a) π-matching network (b) T-matching network

Figure 6 : Three element matching networks

π-section matching network

A π-section matching network consists of 3 elements arranged as follows:

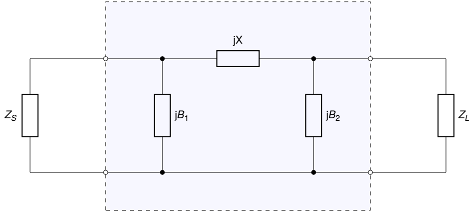


Figure 7 : π-section matching network

The π-Section can be considered as consisting of a type 1 L-section and type 2 L-section in cascade as shown below, where $X = X_1 + X_2$:

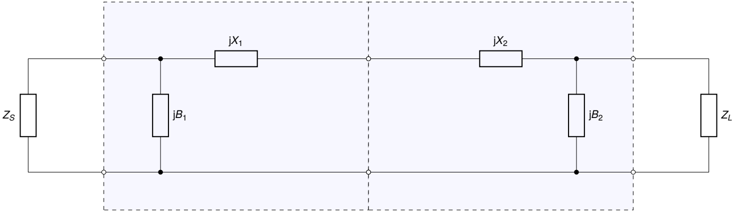


Figure 8 : π-section matching network

The π-section matching network

- The π-section can be considered as consisting of a type 2 L-section followed by a type 1 L-section in cascade as shown in figure 9, where the central element, X, has been split into two reactive elements of the same type, i.e. $X = X_1 + X_2$.
- We also presuppose a "invisible" load resistance, R_x, interposed between the two L sub-networks in figure 9.
- The purpose of L-section 1, therefore, is to match the source to R_x. Similarly, the purpose of L-section 2 is to match R_x to the load. The individual L-sections can be designed according to the principles set out in the previous section, provided we know the value of R_x, that is.
- The value of R_x can be chosen arbitrarily, but it should be smaller than both R_S and R_L, since it is connected to the series arms of the two L-sections[1]. If we start with a required value of Q, however, this will determine the choice of R_x.

The π-section matching network

Consider the deconstructed π-section matching network shown in figure 9. Applying (4) we get the loaded Q of L-network 1, which matches R_S to R_x. We have already stipulated that R_x must be smaller that R_S so we have:

$$Q_{L1} = \sqrt{\left(\frac{R_S}{R_x} - 1\right)} \tag{18}$$

Applying the same logic, the loaded Q of L-network 2, which matches R_x to R_L (which is larger than R_x), is given by :

$$Q_{L2} = \sqrt{\left(\frac{R_L}{R_x} - 1\right)} \tag{19}$$

Since the loaded Q of the overall circuit is determined by the branch of the circuit having the highest loaded Q value, we can write the overall Q of the circuit in figure 9, Q_π, as[1] :

$$Q_\pi = \sqrt{\left(\frac{R_{high}}{R_x} - 1\right)} \tag{20}$$

Where R_{high} is the larger of R_S and R_L. By inspection of (18) to (20) we can see that the overall Q of the π-section will be equal to the highest Q of the two constituent L-sections.

The π-section matching network design

With known values of Q_{L1}, Q_{L2} and R_x we proceed with the π-section design by designing each constituent L-section, using the techniques set out in section **??**, and then combining the central elements.

The reader will note that we have so far only considered the resistive parts of the source and load, R_S and R_L. This is because any reactive parts of the source and load can be absorbed into the parallel branches of the π-section; B_1 and B_2.

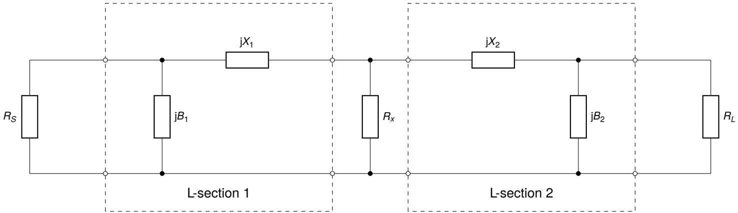


Figure 9 : π-section matching network as a cascade of two L networks

π-network matching example

As an example, we will design a π-section matching network having a Q of 4, to match an antenna having an impedance of $20 + j25\Omega$ at 2GHz to a 50 Ω transmission line. The matching network should be able to pass DC current.

Since the matching network should be able to pass DC current, we need to use the topology shown in figure 10.

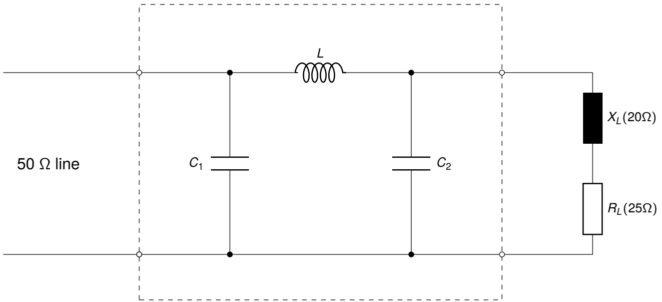


Figure 10 : Π-section matching network example

π-network matching example

Since we are using a π-section, it is more convenient for us to work with an equivalent load admittance, as we need to absorb the reactive part of the load into the π-section element jB_2. We therefore convert the load to an equivalent parallel configuration, so that we can simply add the susceptances. We therefore calculate the load admittance as :

$$Y_L = \frac{1}{20 + j25} = 0.0195 - j0.0244S$$

We can now redraw figure 10 broken down into two L-sections in figure 11, which also shows the virtual resistance, R_x.

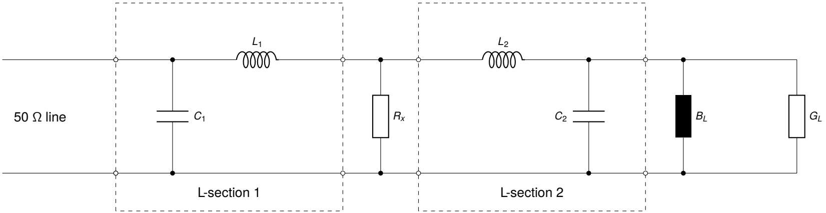


Figure 11 : π-section matching network example

π-network matching example

Assuming we absorb the equivalent load susceptance, B_L, into C_2 of the π-section, the equivalent purely resistive load then becomes :

$$R_L = \frac{1}{G_L} = \frac{1}{0.0195} = 51.28\Omega$$

We can now calculate the value of R_x using (**??**) :

$$R_x = \frac{R_{high}}{Q_\pi^2 + 1} = \frac{51.28}{4^2 + 1} = 3\Omega$$

We now employ the procedures learned in section **??** to design the two constituent L-sections in figure 11. Since $R_L > R_S$ we know that the L-section comprising L_2 and C_2, has the highest Q of the two, i.e. :

$$Q_{L2} = Q_\pi = 4$$

π-network matching example

We now calculate the shunt susceptance, B_2, from (5), remembering to compensate for the negative load susceptance, B_L, by adding an equivalent positive susceptance to B_2 :

$$B_2 = \frac{4}{51.28} + 0.0244 = 0.078 + 0.0244 = 0.1024\mho$$

We can now calculate the capacitance C_2 as follows :

$$C_2 = \frac{B_2}{\omega} = \frac{0.1024}{2\pi \times 2 \times 10^9} = 8.15pF$$

We now apply equation (**??**) to determine the reactance of L_2 :

$$X_2 = \frac{QR_L}{1 + Q_\pi^2} = \frac{4 \times 51.28}{1 + 4^2} = 12\Omega$$

So, the inductance, L_2, is calculated as :

$$L_2 = \frac{X_2}{\omega} = \frac{12}{2\pi \times 2 \times 10^9} = 0.96nH$$

π-network matching example

We now repeat the above design procedure for the L-section comprising L_1 and C_1. We firstly need to calculate Q_{L1} using (18) :

$$Q_{L1} = \sqrt{\left(\frac{50}{3} - 1\right)} = 3.96$$

$$B_1 = \frac{Q_{L1}}{R_S} = \frac{3.95}{50} = 0.079\ S$$

$$C_1 = \frac{B_1}{\omega} = \frac{0.079}{2\pi \times 2 \times 10^9} = 6.30pF$$

$$X_1 = \frac{Q_{L1} R_S}{1 + Q_{L1}^2} = \frac{3.96 \times 50}{1 + 3.96^2} = 11.87\Omega$$

So, the inductance, L_1, is calculated as :

$$L_1 = \frac{X_1}{\omega} = \frac{11.87}{2\pi \times 2 \times 10^9} = 0.945nH$$

π-network matching example

By combining L_1 and L_2 to give $L = 1.90nH$, and removing R_x, we arrive at the final configuration shown in figure 12 :

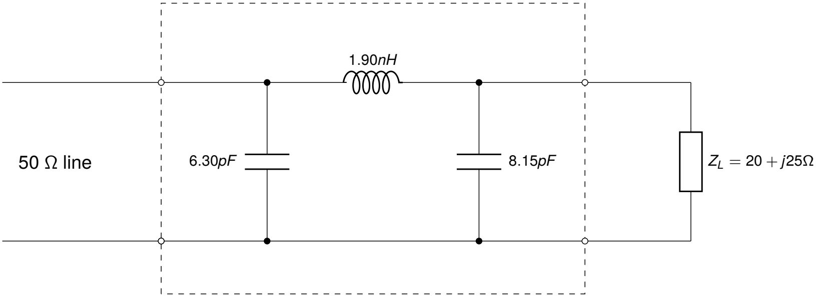
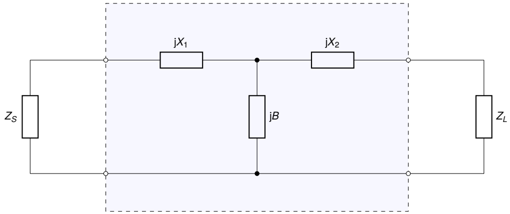


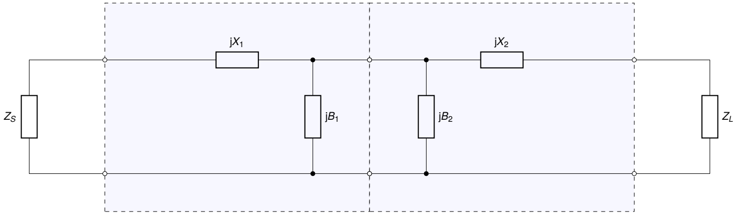
Figure 12 : π-section matching network example - final configuration

T-section matching network

A T-section matching network consists of 3 elements arranged as follows:



The T-Section can be considered as consisting of a type 2 L-section and type 1 L-section in cascade as shown below, where $B = B_1 + B_2$:



T-network matching example

As an example, we will design a T-section matching network having a Q of 4, to match an antenna having an impedance of 20+j25 Ω at 2GHz to a 50 Ω transmission line. This matching network should be able to pass DC current.

We now proceed to design the type 2 L-section comprising L_2 and C_2, shown in figure 13.

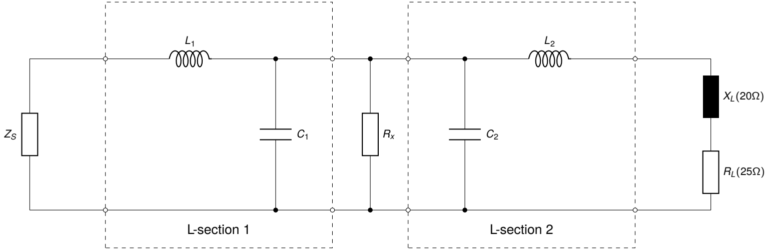


Figure 13 : T-section matching network example

T-network matching example

Since $R_L < Z_o$ in this case, the Q of the L-section C_2, L_2 is equal to the overall Q of the T-section, i.e. :

$$Q_{L2} = Q_T = 4$$

We can now calculate the shunt susceptance, B_2, from (5), noting that the 'load', in this case, is R_x :

$$B_2 = \frac{Q_{L2}}{R_x} = \frac{4}{340} = 0.0118S$$

The value of C_2 can now be calculated as :

$$C_2 = \frac{B_2}{\omega} = \frac{0.0118}{2\pi \times 2 \times 10^9} = 0.94pF$$

T-network matching example

Since the matching network should be able to pass DC current, we need to use the topology shown in figure 14.

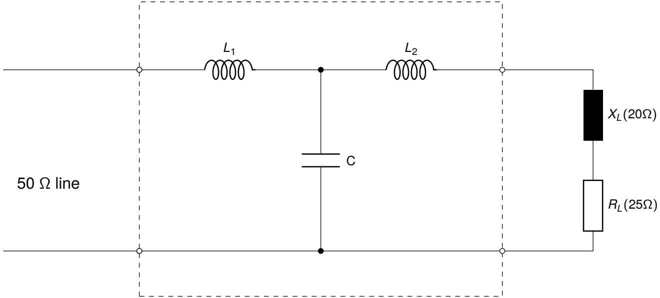


Figure 14 : T-section matching network example

T-network matching example

For clarity, we can redraw figure 14 broken down into two L-sections in figure 13, which shows the virtual resistance, R_x, the value of which can be calculated using (**??**) :

$$R_x = 20\left(4^2 + 1\right) = 340\Omega$$

We now apply equation (**??**) to determine the series reactance X_2, noting that we need to subtract the inductive reactance of the load, X_L :

$$X_2 = \frac{Q_{L2}R_x}{1 + Q_{L2}^2} - X_L = \frac{4 \times 340}{1 + 4^2} - 25 = 55\Omega$$

We can now calculate L_2 as :

$$L_2 = \frac{X_2}{\omega} = \frac{55}{2\pi \times 2 \times 10^9} = 4.38nH$$

T-network matching example

We now repeat the above design procedure for the type 1 L-section comprising L_1 and C_1 to give :

$$Q_{L1} = \sqrt{\left(\frac{340}{50}\right) - 1} = 2.41$$

$$B_1 = \frac{Q_{L1}}{R_x} = \frac{2.41}{340} = 0.0071\ S$$

$$C_1 = \frac{B_1}{\omega} = \frac{0.0071}{2\pi \times 2 \times 10^9} = 0.56pF$$

$$X_1 = \frac{Q_{L1} R_x}{1 + Q^2} = \frac{2.41 \times 340}{1 + 2.41^2} = 120.4\Omega$$

We can now calculate L_1 as :

$$L_1 = \frac{X_1}{\omega} = \frac{120.4}{2\pi \times 2 \times 10^9} = 9.58nH$$

T-network matching example

We now complete the design by removing R_x and combining C_1 and C_2 to give $C = 1.5pF$. The final design is shown in figure 15.

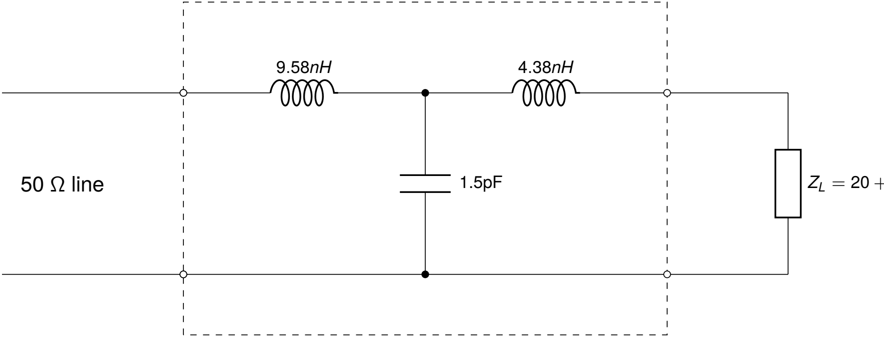


Figure 15 : T-section matching network example - final configuration

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Bandwidth of lumped element matching networks

The bandwidth of any lumped element matching network can be determined from the loaded Q of the circuit, Q_L, by applying (**??**), i.e.:

$$\Delta f = \frac{f_o}{Q_L} \tag{21}$$

In the case of L-sections, the Q and therefore the bandwidth, are solely a function of the load and source resistances. The bandwidth of an L-section can therefore be determined by applying (4) as follows:

$$\Delta f = \frac{f_o}{\sqrt{\left(\frac{R_{high}}{R_{low}}\right) - 1}} \tag{22}$$

The advantage of the three element networks, (T and π), by contrast, is that Q can be chosen, to some extent, as an independent design parameter. This means that we have some degree of choice of bandwidth, independent of the load and source resistances, provided that the chosen Q is larger than that which is available with an L network. This means that T or π-networks are only really suitable for narrow-band applications.

If wider bandwidth are required, a matching network based on cascaded L-sections may be used.

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T to π transformation

The well known *Star-Delta transformation* allows us to convert any given π-network of generalised impedances into an equivalent T-network and vice versa. With reference to figure 16 we can write the following:

T to π transformation :

$$Z_a = \frac{(Z_1Z_2 + Z_1Z_3 + Z_2Z_3)}{Z_2} \quad (23)$$

$$Z_b = \frac{(Z_1Z_2 + Z_1Z_3 + Z_2Z_3)}{Z_1} \quad (24)$$

$$Z_c = \frac{(Z_1Z_2 + Z_1Z_3 + Z_2Z_3)}{Z_3} \quad (25)$$

π to T transformation :

$$Z_1 = \frac{Z_aZ_c}{Z_a + Z_b + Z_c} \quad (26)$$

$$Z_2 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c} \quad (27)$$

$$Z_3 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c} \quad (28)$$

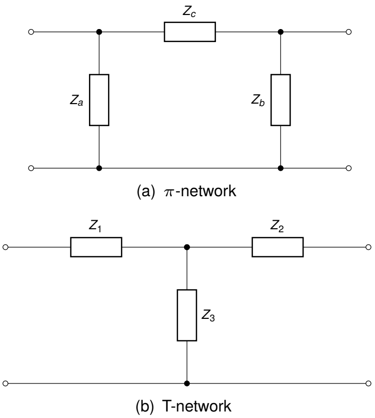

Figure 16 : π to T transformation

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