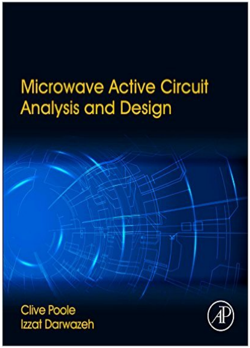
Lecture 16 - Low Noise Oscillator Design

Microwave Active Circuit Analysis and Design

Clive Poole and Izzat Darwazeh

Academic Press Inc.



Intended Learning Outcomes

- Knowledge
  - Understand the nature and impact of phase noise in oscillators.
  - Be familiar with the origin, application and limitations of Leeson's equation.
  - Be familiar with analytical phase noise models for both feedback and negative resistance oscillators, and be able to compare the two models.
  - Understand how a simple phase noise model implies certain design guidelines for low noise oscillators and be able to apply these guidelines.
  - Be familiar with various phase noise measurement techniques.
- Skills
  - Be able to select suitable transistors and resonators for use in low noise oscillator design.
  - Be able to design an oscillator with low phase noise based on the tools and techniques presented.
  - Be able to carry out basic phase noise measurements.

Table of Contents

Definition of phase noise

An ideal oscillator will produce only one sinusoidal frequency component at the nominal centre frequency. Real world oscillators produce a signal which is more complex, containing other, unintended, frequency components. We need a figure of merit by which to measure the purity of a given oscillator output.
The signal output from an ideal oscillator can be described as :

$$V(t) = cos(\omega_o t) \tag{1}$$

Where ω_o is the centre frequency and the amplitude is assumed to be unity. The output of a real oscillator output will contain both amplitude and phase noise components. We can represent the output of a real oscillator as :

$$V(t) = [1 + e(t)]cos(\omega_o t + \Phi(t)) \tag{2}$$

Where $\Phi(t)$ is the time varying phase component and $e(t)$ is the amplitude noise component added to the nominal unit signal amplitude. Due to the inherent amplitude limiting mechanism already described, $e(t)$ will be significantly attenuated, so that $[1 + e(t)] \approx 1$. Nevertheless, the presence of both amplitude and phase fluctuations cause sidebands in the output voltage spectrum of the oscillator. Figure 1 shows the difference in the output spectrum of an ideal oscillator with that of a real oscillator.

Definition of phase noise

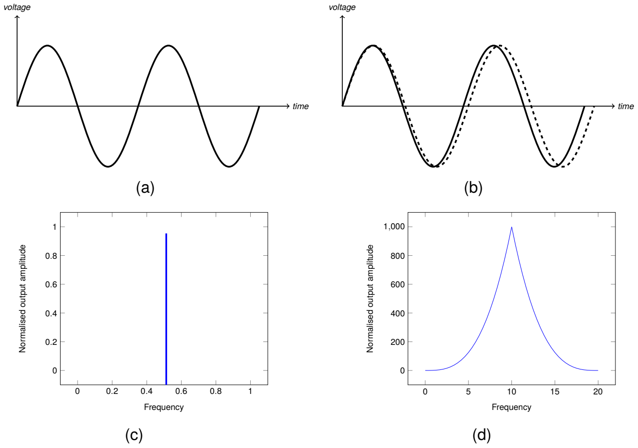


Figure 1 : Ideal and real oscillator output signals and spectra : (a) Ideal oscillator signal, (b) Real oscillator signal, (c) Ideal oscillator spectrum, (d) Real oscillator spectrum

Definition of phase noise

- The US National Institute of Standards and Technology defines single sideband (SSB) phase noise as the ratio of the spectral power density in a 1 Hz bandwidth, measured at an offset frequency from the carrier to the total power of the carrier signal, as illustrated in figure 2.
- The symbol used to represent this quantity is $\mathscr{L}\{\Delta f\}$, which is pronounced "script *L* of delta *f*", or sometimes simply $\mathscr{L}\{f_m\}$ ("script *L* of *f m*"), in cases where f_m is taken to represent the frequency offset.

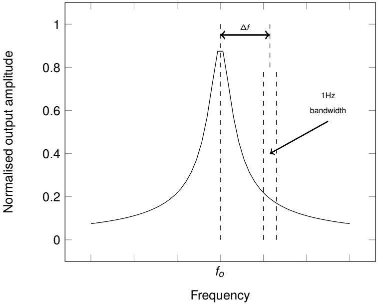


Figure 2 : Noise at offset from a carrier

The basic definition of $\mathscr{L}\{\Delta f\}$, which is usually expressed in decibels, is as follows :

$$\mathscr{L}\{\Delta f\}_{dB} = 10\ log\left[\frac{\text{Power Spectral Density in 1 Hz bandwidth at } (f_o + \Delta f)}{P_{carrier}}\right] \quad (3)$$

Typical phase noise profile

The unit of phase noise is dBc/Hz (dB relative to carrier frequency, per 1 Hz offset). A typical phase noise plot is shown in figure 3, which shows four distinct regions, in order of distance from the carrier :

(i) An initial section, close to the carrier, which has a slope of $1/f^3$ (-30dB/decade).

(ii) A second section having a slope of $1/f^2$ (-20dB/decade).

(iii) A third section having a slope of $1/f$ (-10dB/decade).

(iv) A final horizontal section corresponding to the thermal noise floor.

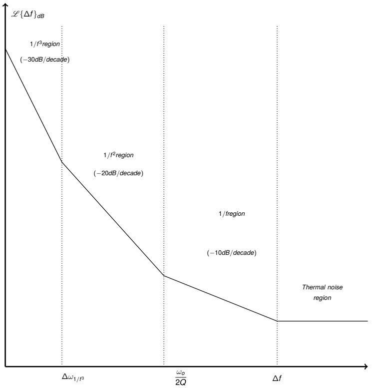


Figure 3 : Typical oscillator phase noise profile

Table of Contents

Why oscillator phase noise is important?

- The ’skirts’ surrounding the centre frequency in figure 2 represent additional radio energy produced by the oscillator outside of its nominal design frequency
- If the oscillator in question is a local oscillator in a transmitter then this unwanted energy may overwhelm nearby weak channels
- Because the phase noise spectral density grows directly with the transmitted signal power, and at a given point in space, the noise sidebands of a strong transmitter may be greater than another faded or attenuated signal occupying the same frequency, as shown in figure 4.

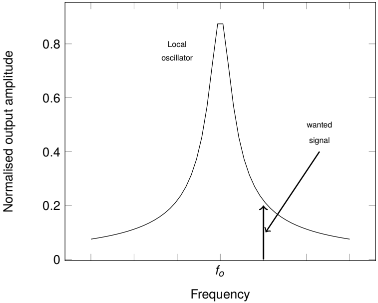


Figure 4 : Interference caused by phase noise

Why oscillator phase noise is important?

Table of Contents

The thermal noise component of phase noise

- Thermal noise is ubiquitous in any electronic system. It can therefore be expected to form an important component of any phase noise in an oscillator.
- In any perfectly resistively matched system at room temperature, which is normally taken as being 290 K which is equivalent to $17^{o}C$, the background thermal noise spectral density level is approximated by:

$$N = k_B T = 1.38 \times 10^{-23} \times 290 = -174 dBm/Hz \tag{4}$$

  This is the minimum level of noise that exists in any 1 Hz bandwidth at room temperature, and is commonly referred to as the *thermal noise floor*
- This thermal noise is composed of both amplitude noise and phase noise components which are assumed to be approximately equal; a concept embodied in the *Equipartition Theorem* [3, 1]. Whilst the treatment of noise in chapter**??** focussed on amplitude noise, the discussion in this chapter focusses on the phase component of the noise
- It is normally assumed that, when talking about oscillators, the amplitude noise portion of the thermal noise floor is suppressed by the amplitude limiting mechanism discussed previously. This means that the thermal phase noise floor is 3 dB below the noise floor given by (8), i.e. :

$$N_{phase} = \frac{k_B T}{2} = -177 dBm/Hz \tag{5}$$

The thermal noise component of phase noise

The active element in an oscillator will also inevitably contribute an additional quantity of noise to the thermal noise floor. The amount of noise added by the amplifier is measured as the ratio of output noise to input noise power, adjusted for the gain of the amplifier. This is quantified in terms of the *noise factor* (F), which, when expressed in dB, is referred to as the *noise figure* (NF). We can thus write the equivalent total thermal phase noise at the input of an amplifier of noise factor F as :

$$N_{phase} = \frac{F k_B T}{2} \tag{6}$$

In terms of dB, (6) can be expressed as :

$$N_{phase(dB)} = 10\, log \left[\frac{F k_B T}{2} \right] = -177 dBm/Hz + (NF)_{dB} \tag{7}$$

If we now apply a signal, of power P_{in}, and a specific frequency, f_o, to the input of the amplifier in question, we can compute the thermal phase noise power in a 1 Hz bandwidth at some frequency offset Δf away from f_o by the ratio :

$$\mathscr{L}\{\Delta f\} = \left(\frac{k_B T F}{2 P_{in}} \right) \tag{8}$$

The reader will notice that the right hand side of equation (8) does not contain term Δf. This is because this equation describes a simplistic case for an oscillator where all the noise is thermal noise having a response which is flat irrespective of offset frequency.

Flicker noise in oscillators

- 1/f noise, sometimes called ’flicker noise, is specific to semiconductor devices.
- The magnitude of flicker noise is inversely proportional to frequency.
- Flicker noise in semiconductor devices is believed to be caused by contamination and crystal defects in pn junctions.

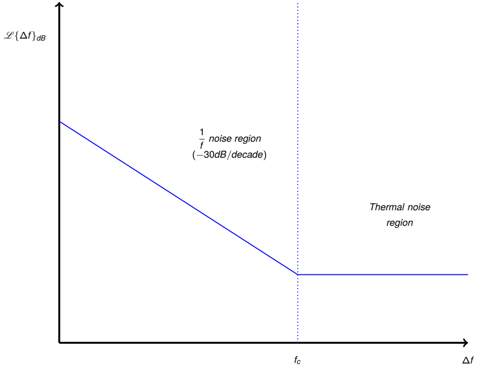


Figure 5 : Phase noise profile with $1/f$ (flicker) noise component

- 1/f noise is directly related to the current density in the transistor.
- Transistors with high I_{cmax} used at low currents have best 1/f performance.
- 1/f noise is especially pronounced in FETs with small channels
- 1/f noise gets upconverted in oscillators and results in noise sidebands either side of the carrier.

Flicker noise in oscillators

- The flicker corner frequency, f_c, is defined at the point at which the flicker noise power equals the underlying thermal noise power. In other words, this is the point at which the total noise power increases by 3dB as we approach f_o from far away
- At this point, to avoid confusion, we will standardise on the use of f_m to represent the offset frequency in Hz distant from the carrier, i.e. the same quantity that we have referred to as Δf up until this point
- From figure 5 we can write an empirical expression for $\mathscr{L}\{f_m\}$ that extends (8) to include the flicker noise component as follows :

$$\mathscr{L}\{f_m\} = \frac{k_B T F}{2P_{in}} \left(1 + \frac{f_c}{f_m}\right) \tag{9}$$

By inspection we can see that, when $f_m >> f_c$, (9) approximates (8)

Table of Contents

Modelling oscillator phase noise

The most well known model of oscillator phase noise is that originally proposed by Professor D. B. Leeson in 1966 [4]. Leeson's model assumes a simple feedback oscillator model shown in figure 6, and is based on the following assumptions:

(i) The amplifier is noiseless, has a high gain and limits at a level corresponding to the nominal output power.

(ii) The resonator is a bandpass type centred at the frequency of oscillation and has a loaded Q of Q_l.

(iii) The noise source represents all noise sources in the oscillator, including those introduced by the amplifier and the resonator.

(iv) The limiting action of the amplifier removes the AM component of the noise, leaving only the phase component.

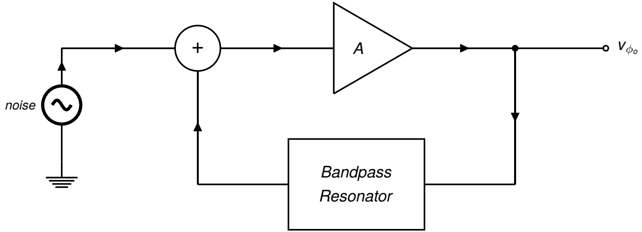


Figure 6 : Leeson's feedback oscillator model

Leeson's model of oscillator phase noise

Based on the simple model of figure 6 Professor Leeson proposed what has come to be known as the *Leeson equation*, which can be stated as follows:

$$\mathscr{L}\{f_m\} = \left(\frac{Fk_BT}{2P_S}\right)\left[1 + \left(\frac{f_0}{2Q_l f_m}\right)^2\right]\left(1 + \frac{f_c}{f_m}\right) \tag{10}$$

where :

F = the 'effective noise factor' of the amplifier.
P_S = the oscillation signal power.
f_o = the oscillator centre frequency.
Q_l = the loaded Q of the resonator.
f_m = the offset frequency from the carrier at which phase noise is measured (in a 1Hz bandwidth).
f_c = the corner frequency between the $1/f^2$ and $1/f^3$ slope region.

Leeson's model of oscillator phase noise

Leeson's equation can be visualised as in figure 7.

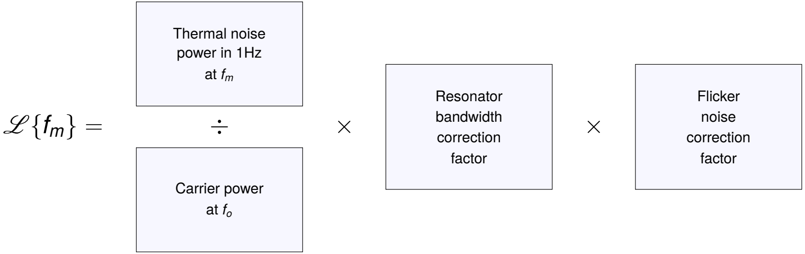


Figure 7 : Diagrammatic representation of Leeson's equation

Leeson's model of oscillator phase noise

- Leeson's equation is essentially an empirical description of the observed phase noise profile shown in figure 8.
- One key message of Leeson's equation is that the spectrum increases as $1/f^2$ when f_m is less than $f_o/2Q$ and as $1/f^3$ when f_m is also less than f_c, which is almost always less than the measured device $1/f$ corner frequency because the modulation conversion does not raise the noise above the thermal floor.

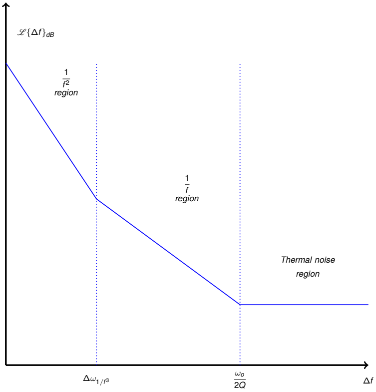


Figure 8 : Phase noise profile according to Leeson's model

Limitations of Leeson's model

1. It assumes that the oscillator operates at the center frequency of the resonator. The fact this condition is rarely met in practice, especially at high frequencies, means that real resonators will be less effective at suppressing phase noise than the Leeson equation predicts.
2. The amplifier will not be matched for maximum power transfer in the input circuit, neither will it be matched for minimum noise factor, making the parameters P_S and F in the Leeson equation difficult to predict.
3. The loaded Q of the resonator is a parameter that is difficult to determine in practice[5].
4. It does not take account of an observed $1/f^3$ phenomenon that results in higher noise density at small frequency offsets from the carrier.
5. Being a linear model, it cannot account for non-linear phenomena, such as the conversion of amplitude modulated noise components into phase modulated noise components, referred to as 'AM-PM conversion' [6].
6. Being a Linear Time Invariant (LTI) model, it is not suitable for modelling some common classes of oscillators, such as relaxation and ring oscillators.

Analytical phase noise models : Feedback oscillator

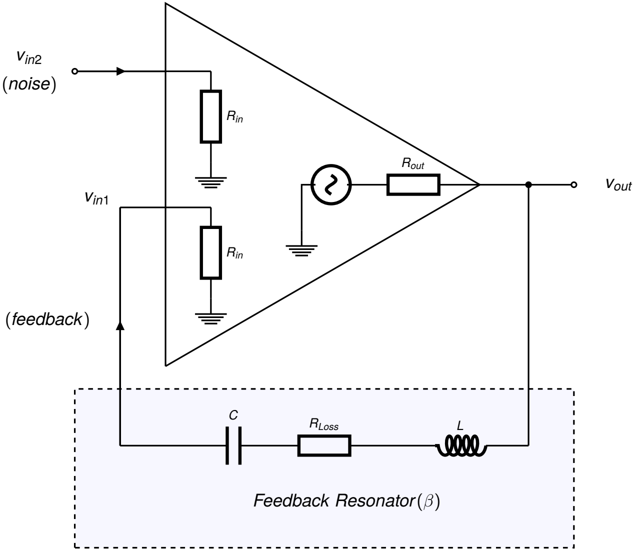


Figure 9 : Oscillator feedback model[1]

Analytical phase noise models : Feedback oscillator

The voltage transfer function of the amplifier in figure 9 is given by :

$$v_{out} = G\left(v_{in1} + v_{in2}\right) = G\left(\beta v_{out} + v_{in2}\right) \tag{11}$$

Where G is the voltage gain of the amplifier between either of the inputs separately and the output, and β is the voltage feedback factor defined by the resonator. In this case we are primarily interested in the effect of the injected noise voltage, V_{in2} on the output. From (11), therefore, we can write :

$$\frac{v_{out}}{v_{in2}} = \frac{G}{1 - (\beta G)} \tag{12}$$

The resonator voltage feedback factor, from output to input 1 of figure 9, can be derived by inspection :

$$\beta = \frac{R_{in}}{R_{out} + R_{loss} + R_{in} + j\left(\omega L - 1/\omega C\right)} \tag{13}$$

We are interested in the behaviour of the resonator at frequencies very close to the carrier, i.e. $\omega = \omega_o \pm \Delta\omega$, where ω_o is the carrier frequency and $\Delta\omega$ is a small frequency offset.

Analytical phase noise models : Feedback oscillator

Let us consider just the imaginary term, $(\omega L - 1/\omega C)$, in the denominator of (13), which can be re-written as follows :

$$(\omega L - 1/\omega C) = \frac{(\omega^2 LC - 1)}{\omega C} \tag{14}$$

Substituting $\omega = \omega_o \pm \Delta\omega$ for ω in (14) gives :

$$\frac{(\omega_o \pm \Delta\omega)^2 LC - 1}{(\omega_o \pm \Delta\omega) C} \tag{15}$$

As we are only interested in very small values of $\Delta\omega$, we can apply the binomial approximation in this specific case, which allows us to write :

$$(\omega_o \pm \Delta\omega)^2 \approx \omega_o \left(\omega_o \mp 2\Delta\omega\right) \tag{16}$$

Applying (18) to (15) gives:

$$\frac{(\omega_o \pm \Delta\omega)^2 LC - 1}{(\omega_o \pm \Delta\omega) C} \approx \frac{\omega_o(\omega_o \mp 2\Delta\omega) LC - 1}{(\omega_o \pm \Delta\omega) C} \tag{17}$$

Analytical phase noise models : Feedback oscillator

As we are only interested in very small values of $\Delta\omega$, we can apply the binomial approximation in this specific case, which allows us to write :

$$(\omega_o \pm \Delta\omega)^2 \approx \omega_o (\omega_o \mp 2\Delta\omega) \tag{18}$$

Applying (18) to (15) gives:

$$\frac{(\omega_o \pm \Delta\omega)^2 LC - 1}{(\omega_o \pm \Delta\omega)C} \approx \frac{\omega_o(\omega_o \mp 2\Delta\omega)LC - 1}{(\omega_o \pm \Delta\omega)C} \tag{19}$$

Noting that $\omega_o^2 = 1/LC$, we can reduce (19) to:

$$\frac{\mp 2\omega_o \Delta\omega L}{(\omega_o \pm \Delta\omega)} \tag{20}$$

Since $\omega_o \gg \Delta\omega$ we can further approximate (20) to simply:

$$\mp 2\Delta\omega L \tag{21}$$

Now, we note that the loaded Q of the resonator, Q_L, is defined by :

$$Q_L = \frac{\omega_o L}{(R_{out} + R_{loss} + R_{in})} \tag{22}$$

Analytical phase noise models : Feedback oscillator

We can now combine (13) with (21) and (22) to give us the feedback factor in terms of $\Delta\omega$ and Q_L, for small values of $\Delta\omega$:

$$\beta = \frac{R_{in}}{(R_{out} + R_{loss} + R_{in})\left[1 \pm 2jQ_L \dfrac{\Delta\omega}{\omega_o}\right]} \tag{23}$$

Let us now consider the feedback factor at the centre frequency (i.e. at resonance), β_o. If we set $\Delta\omega = 0$ in (23) we then have :

$$\beta_o = \frac{R_{in}}{R_{out} + R_{loss} + R_{in}} \tag{24}$$

Which accords with our understanding that the net reactance of the resonator, $(\omega L - 1/\omega C)$, will be zero at resonance, resulting in a purely real value of β_o. Since the unloaded Q of the resonator, Q_o, is simply :

$$Q_o = \frac{\omega_o L}{R_{loss}} \tag{25}$$

Analytical phase noise models : Feedback oscillator

We can define the ratio :

$$\frac{Q_L}{Q_o} = \frac{R_{loss}}{R_{out} + R_{loss} + R_{in}} \tag{26}$$

Which implies :

$$\left[1 - \frac{Q_L}{Q_o}\right] = \frac{R_{out} + R_{in}}{R_{out} + R_{loss} + R_{in}} \tag{27}$$

Now, by employing (27) in (24) we can write :

$$\beta_o = \frac{R_{in}}{R_{out} + R_{in}} \left[1 - \frac{Q_L}{Q_o}\right] \tag{28}$$

With reference to (23), and replacing angular frequency by $\omega = 2\pi f$, we can write the overall resonator response in terms of β_o as :

$$\beta = \beta_o \left[\frac{1}{1 \pm 2jQ_L \dfrac{\Delta f}{f_o}}\right] \tag{29}$$

Or in its fuller form as :

$$\beta = \frac{R_{in}}{R_{out} + R_{in}} \left[1 - \frac{Q_L}{Q_o}\right] \left[\frac{1}{1 \pm 2jQ_L \dfrac{\Delta f}{f_o}}\right] \tag{30}$$

Analytical phase noise models : Feedback oscillator

We are now in a position to define the voltage transfer characteristic of the amplifier in figure 9 by incorporating (30) into (12), thus :

$$\frac{V_{out}}{V_{in2}} = \frac{G}{1 - G\left[\frac{R_{in}}{R_{out} + R_{in}}\right]\left[1 - \frac{Q_L}{Q_o}\right]\left[\frac{1}{1 \pm 2jQ_L\frac{\Delta f}{f_o}}\right]} \tag{31}$$

Under steady state oscillation conditions the voltage gain of the amplifier, at $f = f_o$, according to the Barkhausen criterion, is defined by :

$$G = \frac{1}{\beta_o} = \frac{1}{\frac{R_{in}}{R_{out} + R_{in}}\left[1 - \frac{Q_L}{Q_o}\right]} \tag{32}$$

Analytical phase noise models : Feedback oscillator

Replacing the term G in (31) by (32) gives :

$$\frac{V_{out}}{V_{in2}} = \frac{G}{1 - \dfrac{1}{1 \pm 2jQ_L \dfrac{\Delta f}{f_o}}} \tag{33}$$

$$= \frac{1}{\left[\dfrac{R_{in}}{R_{out} + R_{in}}\right]\left[1 - \dfrac{Q_L}{Q_o}\right]\left[1 - \dfrac{1}{1 \pm 2jQ_L \dfrac{\Delta f}{f_o}}\right]} \tag{34}$$

The Q multiplication process causes the noise to fall to the thermal noise floor within the 3dB bandwidth of the resonator[1]. The noise of interest therefore occurs within the boundaries of $Q_l(\Delta f/f_o) << 1$, so we can apply the binomial approximation to (33) resulting in a further simplification :

$$\frac{V_{out}}{V_{in2}} = \frac{G}{\pm 2jQ_L \dfrac{\Delta f}{f_o}} \tag{35}$$

$$= \frac{1}{\left[\dfrac{R_{in}}{R_{out} + R_{in}}\right]\left[1 - \dfrac{Q_L}{Q_o}\right]\left[\pm 2jQ_L \dfrac{\Delta f}{f_o}\right]} \tag{36}$$

Analytical phase noise models : Feedback oscillator

The gain of the feedback system has been incorporated into (35) in terms of Q/Q_o, since the gain is set by the insertion loss of the resonator.
We now turn our attention to the thermal noise input voltage at input 2 of the amplifier in figure 9, which we can write as :

$$V_{in2} = \sqrt{4k_B TBR_{in}} \tag{37}$$

Where k_B is Boltzmann's constant, T is the absolute temperature and B is the bandwidth of interest. Since we are interested in the ratio of noise to signal power, we will deal in terms of squared voltages. If the source resistance is equal to R_{in}, the total noise power available at the input of the amplifier is equal to $k_B TB$.

Since we are specifically interested in noise power in a 1 Hz bandwidth at an offset Δf from the carrier, we set $B = 1$. We relate the noise power at the amplifier input to that at the output via the amplifier noise factor, F.

Analytical phase noise models : Feedback oscillator

If we apply the above reasoning to (35) we can write the square of the output voltage in a 1 Hz bandwidth at an offset, f_m, from the carrier as :

$$(V_{out}(f_m))^2 = \frac{F k_B T R_{in}}{4Q_L^2 \left[\frac{R_{in}}{R_{out} + R_{in}}\right]^2 \left[1 - \frac{Q_L}{Q_o}\right]^2} \left(\frac{f_o}{f_m}\right)^2 \tag{38}$$

Since Q_o is fixed by the type of resonator, but the ratio (Q_L/Q_o) can be varied by adjusting the resonator coupling, we can rewrite (38) in a more useful form that separates constants and variables, thus :

$$(V_{out}(f_m))^2 = \frac{F k_B T R_{in}}{4Q_o^2 \left(\frac{Q_L}{Q_o}\right)^2 \left[\frac{R_{in}}{R_{out} + R_{in}}\right]^2 \left[1 - \frac{Q_L}{Q_o}\right]^2} \left(\frac{f_o}{f_m}\right)^2 \tag{39}$$

Analytical phase noise models : Feedback oscillator

Equation (39) incorporates both amplitude and phase components of the input noise signal. In practice, the amplitude fluctuations are largely suppressed by the amplitude limiting function of the amplifier. This has the effect of halving the total input noise power defined in (38). A more correct equation for the square of the output voltage is therefore :

$$(V_{out}(f_m))^2 = \frac{Fk_B TBR_{in}}{8Q_o^2 \left(\frac{Q_L}{Q_o}\right)^2 \left[\frac{R_{in}}{R_{out}+R_{in}}\right]^2 \left[1-\frac{Q_L}{Q_o}\right]^2} \left(\frac{f_o}{f_m}\right)^2 \quad (40)$$

The generally accepted definition of phase noise is the ratio of output noise power in a 1 Hz bandwidth at a frequency offset f_m to the total output power. If the total output power is $V_{out\ max\ rms}$, then we can write :

$$\mathscr{L}\{f_m\} = \frac{(V_{out}(f_m))^2}{(V_{out\ max\ rms})^2} \quad (41)$$

Applying (40) gives :

$$\mathscr{L}\{f_m\} = \frac{Fk_B TR_{in}}{8Q_o^2 \left(\frac{Q_L}{Q_o}\right)^2 \left[\frac{R_{in}}{R_{out}+R_{in}}\right]^2 \left[1-\frac{Q_L}{Q_o}\right]^2 (V_{out\ max\ rms})^2} \left(\frac{f_o}{f_m}\right)^2 \quad (42)$$

Analytical phase noise models : Feedback oscillator

P_{RF} is limited by the maximum voltage swing at the output of the amplifier and the value of $R_{out} + R_{loss} + R_{in}$, i.e. :

$$P_{RF} = \frac{(V_{out\ max\ rms})^2}{R_{out} + R_{loss} + R_{in}} \tag{43}$$

Equation (42) now becomes :

$$\mathscr{L}\{f_m\} = \frac{Fk_BT(R_{out} + R_{in})^2}{8Q_o^2\left(\frac{Q_L}{Q_o}\right)^2 R_{in}\left[1 - \frac{Q_L}{Q_o}\right]^2 P_{RF}(R_{out} + R_{loss} + R_{in})}\left(\frac{f_o}{f_m}\right)^2 \tag{44}$$

We note that :

$$\frac{R_{out} + R_{in}}{R_{out} + R_{loss} + R_{in}} = \left[1 - \frac{Q_L}{Q_o}\right] \tag{45}$$

So, the ratio of sideband noise in a 1 Hz bandwidth at an offset Δf to the total power given in (44) therefore becomes :

$$\mathscr{L}\{f_m\} = \frac{Fk_BT}{8Q_o^2\left(\frac{Q_L}{Q_o}\right)^2\left[1 - \frac{Q_L}{Q_o}\right] P_{RF}}\left[\frac{R_{out} + R_{in}}{R_{in}}\right]\left(\frac{f_o}{f_m}\right)^2 \tag{46}$$

Everard outlines three possible cases, as follows :

Case 1 : High efficiency oscillator

If $R_{out} \approx 0$, as would be the case for any high efficiency oscillator then (46) simplifies to :

$$\boxed{\mathscr{L}\{f_m\} = \frac{F k_B T}{8Q_o^2 \left(\frac{Q_L}{Q_o}\right)^2 \left[1 - \frac{Q_L}{Q_o}\right] P_{RF}} \left(\frac{f_o}{f_m}\right)^2} \tag{47}$$

Case 2 : Microwave amplifier

If $R_{out} = R_{in}$, as would be the case for most microwave amplifiers then (46) simplifies to :

$$\mathscr{L}\{f_m\} = \frac{Fk_BT}{4Q_o^2\left(\frac{Q_L}{Q_o}\right)^2\left[1-\frac{Q_L}{Q_o}\right]P_{RF}}\left(\frac{f_o}{f_m}\right)^2 \tag{48}$$

$$P_{AVO} = \frac{(V_{out\ max\ rms})^2}{4R_{out}} \tag{49}$$

Equation (46) then becomes :

$$\mathscr{L}\{f_m\} = \frac{Fk_BTR_{in}}{8Q_o^2\left(\frac{Q_L}{Q_o}\right)^2\left[\frac{R_{in}}{R_{out}+R_{in}}\right]^2\left[1-\frac{Q_L}{Q_o}\right]^2 P_{AVO}(4R_{out})}\left(\frac{f_o}{f_m}\right)^2 \tag{50}$$

which can be rearranged as :

$$\boxed{\mathscr{L}\{f_m\} = \frac{Fk_BTR_{in}}{32Q_o^2\left(\frac{Q_L}{Q_o}\right)^2\left[1-\frac{Q_L}{Q_o}\right]^2 P_{AVO}}\left[\frac{(R_{out}+R_{in})^2}{R_{out}R_{in}}\right]^2\left(\frac{f_o}{f_m}\right)^2} \tag{51}$$

Case 3 : Matched output

The term :

$$\frac{(R_{out}+R_{in})^2}{R_{out}R_{in}} \tag{52}$$

attains a minimum value of four when $R_{out} = R_{in}$. In this case (46) becomes :

$$\boxed{\mathscr{L}\{f_m\} = \frac{Fk_BT}{8(Q_o)^2\left(\frac{Q_L}{Q_o}\right)^2\left(1-\frac{Q_L}{Q_o}\right)^2 P_{AVO}}\left(\frac{f_o}{f_m}\right)^2} \tag{53}$$

Feedback oscillator phase noise model : summary

A general equation can be written which describes all three cases :

$$\mathscr{L}\{f_m\} = A \frac{F k_B T}{8(Q_o)^2 \left(\frac{Q_L}{Q_o}\right)^2 \left(1 - \frac{Q_L}{Q_o}\right)^N P} \left(\frac{f_o}{f_m}\right)^2 \qquad (54)$$

Where the parameters A and N are determined as follows :

1) N = 1 and A = 1 if P is defined as P_{RF} and R_{OUT} = 0 (47).
2) N = 1 and A = 2 if P is defined as P_{RF} and R_{OUT} = R_{IN} (51).
3) N = 2 and A = 1 if P is defined as P_{AVO} and R_{OUT} = R_{IN} (53).

If we take assumption 3, equation (54) becomes :

$$\mathscr{L}\{f_m\} = 1 \frac{F k_B T}{8(Q_o)^2 \left(\frac{Q_L}{Q_o}\right)^2 \left(1 - \frac{Q_L}{Q_o}\right)^2 P_{AVO}} \left(\frac{f_o}{f_m}\right)^2 \qquad (55)$$

Analytical phase noise models : Negative resistance oscillato

The analytical model of phase noise for a feedback oscillator can be extended to cover the negative resistance oscillators also. Once again, we will follow Professor Everard's analysis[2] here, with reference to figure 10.

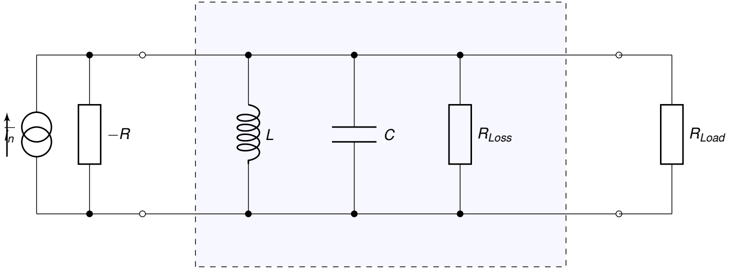


Figure 10 : Negative resistance oscillator model[2]

rison of feedback and negative resistance oscillator phase noise

It is interesting to compare the phase noise equation for a negative-resistance oscillator (**??**) with the phase noise equation for a feedback oscillator (**??**), as follows :

Feedback Oscillator	Negative Resistance Oscillator
$\mathscr{L}\{f_m\} = \dfrac{2k_B T}{Q_o^2 P_{AVO}} \left(\dfrac{f_o}{f_m}\right)^2$	$\mathscr{L}\{f_m\} = \dfrac{k_B T}{2Q_o^2 P} \left(\dfrac{f_o}{f_m}\right)^2$

- The equation for a feedback oscillator is four times larger than that for the negative resistance oscillator.
- A difference of a factor of two can be explained by the fact that P in the negative resistance is the power dissipated in the resonator, whereas P_{AVO} is twice the value of the power dissipated in the resonator under optimum operating conditions. The other factor of two is probably due to the fact that the SNR is set by the power at the input of the amplifier, which is $1/4$ in the case of the feedback oscillator.
- It may still, therefore, be important in some circumstances (crystal oscillators, for example) in which the power in the resonator must be kept low, that the negative-resistance oscillator improves the phase noise by a factor of two for the same power dissipated in the resonator[2].

Table of Contents

Low Noise Oscillator Design

Two key elements of low phase-noise oscillator design :

1. A resonant circuit with a high Q-factor
2. Low noise design of the active circuit

Low Noise Oscillator Design : Resonator

1. Reduce noise by maximizing the reactive energy by means of a high RF voltage across the resonator. Use a low LC ratio.
2. To construct a resonant structure with a high Q-factor low losses are required in all of the constituent parts, such as :
   - Q of resonator device itself
   - Series resistance of capacitors
   - Series resistance of any tuning diodes
   - Loss of printed circuit board

Low Noise Oscillator Design : Transistor

1. Low 1/f noise of the transistor in the oscillator is very important, because the 1/f noise appears as sideband noise around the carrier frequency of the oscillator output signal.
2. The basic rules to select the right transistor for an optimized design are:
   - The best oscillator transistor is a device with the lowest possible fT. A commonly used criteria is: $f_T \leq 2xf_{osz}$.
   - The 1/f noise is directly related to the current density in the transistor. Transistors with high Icmax used at low currents have best 1/f performance.
   - However, the f_T of a transistor drops as current decreased. Additionally, the parasitic capacitances of a high current transistors are higher due to the larger transistor geometry.

Table of Contents

The direct method

The simplest and most intuitive method of measuring phase noise is to connect the oscillator to a high quality spectrum analyser, as shown in figure 11. The power of the carrier is measured and a measurement of the power spectral density of the oscillator noise, at a specified offset frequency, is also made and referenced to the carrier power level.

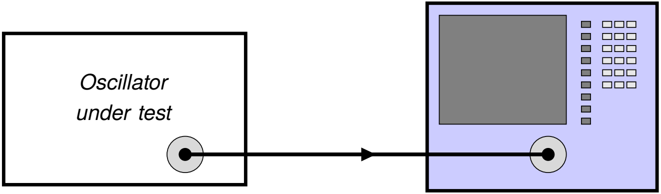


Figure 11 : Phase noise measurement : direct method

Although the direct method of measurement is simple and uses a readily available piece of test equipment (the spectrum analyser), it has some serious limitations.

The direct method

The biggest limiting factor is the quality of the spectrum analyser being used, but all spectrum analysers share some common limitations, as follows :

1. The 3 dB bandwidth and the noise bandwidth of the analyser's resolution bandwidth filters are not identical and correction factors must be used.
2. There are errors associated with the way the peak detector inside most spectrum analysers responds to noise, meaning that RMS noise power will be under-reported by a factor of 1.05 dB. In addition, the logging process in spectrum analysers tend to amplify noise peaks less than the rest of the noise signal resulting in a reported power that is less than the actual noise power. Combining these two effects results in a noise power measurement that is 2.5 dB below the actual noise power.
3. The noise floor of the spectrum analyser and the residual FM of the analyser's own local oscillator will limit the accuracy of the measurement.
4. Lastly, spectrum analysers generally only measure the scalar magnitude of noise sidebands of the signal and are not able to differentiate between amplitude noise and phase noise. Finally, the measurement process using a spectrum analyser involves having to make a noise measurement at each frequency offset of interest. This will be very time consuming if done manually, although it can be automated if the spectrum analyser is programmable.

The phase detector method

- The basic phase detector method is shown in figure 12
- Two sources, at the same frequency and in phase quadrature, are presented to a double-balanced mixer which, together with a low pass filter, acts as a phase detector.
- The difference frequency emerging from the low pass filter has an average voltage level of 0V. Riding on this DC signal are AC voltage fluctuations proportional to the combined phase noise of the two sources. The baseband signal is amplified and then fed into a baseband spectrum analyser.
- In order for this method to work, phase quadrature between the oscillator under test and the reference oscillator must be strictly maintained. This is achieved by making the frequency of the reference oscillator electronically tunable and driving this from a quadrature detector connected to the output of the phase detector.

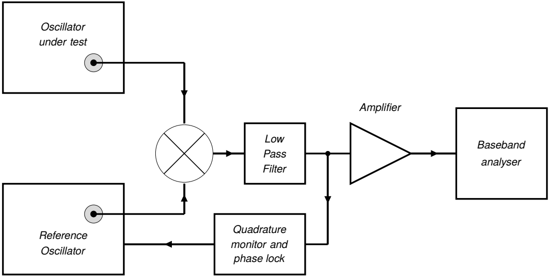


Figure 12 : Phase noise measurement : Phase detector method

The delay line/frequency discriminator method

- In contrast to the phase detector method, the frequency discriminator method of phase noise measurement has the advantage of not requiring a reference source phase locked to the oscillator under test, and is therefore somewhat simpler and lower cost to implement
- The key element in the measurement set-up is an analogue delay line, as shown in figure 13
- Short term frequency fluctuations in the oscillator under test are converted into voltage fluctuations that can be measured by a baseband analyser. This process is accomplished in two stages: first, frequency fluctuations are converted into phase fluctuations and then these phase fluctuations are converted into voltage fluctuations

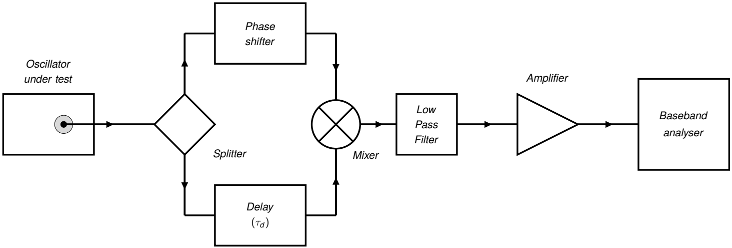


Figure 13 : Phase noise measurement : Frequency discriminator method

References

J Everard.
Fundamentals of RF circuit design: with low noise oscillators.
John Wiley and Sons Ltd., Chichester, 2001.

J. Everard, Min Xu, and S. Bale.
Simplified phase noise model for negative-resistance oscillators and a comparison with feedback oscillator models.
Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on, 59(3):382–390, 2012.

T H Lee and A Hajimiri.
Oscillator phase noise a tutorial.
Solid-State Circuits, IEEE Journal of, 35(3):326–336, 2000.

D B Leeson.
A simple model of feedback oscillator noise spectrum.
Proceedings of the IEEE, 54(2):329–330, 1966.

J-C Nallatamby, M. Prigent, M. Camiade, and J.J. Obregon.
Extension of the leeson formula to phase noise calculation in transistor oscillators with complex tanks.
Microwave Theory and Techniques, IEEE Transactions on, 51(3):690–696, March 2003.

Akio Takaoka and Katsumi Ura.
Noise analysis of nonlinear feedback oscillator with AM-pm conversion coefficient.
Microwave Theory and Techniques, IEEE Transactions on, 28(6):654–662, June 1980.