Lecture 4 - The Smith Chart

Microwave Active Circuit Analysis and Design

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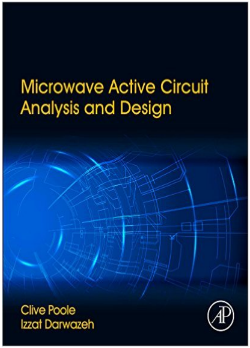

Intended Learning Outcomes

- Knowledge
  - Understand the derivation of the Smith Chart based on a *bilinear transformation*.
  - Understand the benefits of the Smith Chart as a circuit design tool.
  - Be aware of the significance of various regions on the Smith Chart in terms of resistance/reactance categories.
  - Understand the meaning and application of constant Q contours on the Smith Chart.
  - Be familiar with variants of the Smith Chart, such as the Compressed Smith Chart, and their applications.
- Skills
  - Be able to plot an impedance or admittance on the Smith Chart and convert to reflection coefficient (and vice versa).
  - Be able to use the Smith Chart to determine the input resistance of a transmission line at a certain distance from a load.
  - Be able to draw constant Q contours on the Smith Chart and apply them in circuit design.

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The Smith Chart

- In the days before electronic calculators, engineers resorted to a variety of graphical calculation devices, or nomograms, to solve complex engineering problems quickly and with an acceptable degree of accuracy (typically up to 3 decimal places).
- The Smith Chart is an example of such a nomogram invented in 1939 by Phillip H. Smith whilst he was working for The Bell Telephone Laboratories.
- The Smith chart is a graphical representation of equation relating impedance and reflection coefficient :

$$\Gamma = \frac{Z - Z_o}{Z + Z_o} \tag{1}$$

Circles of constant resistance

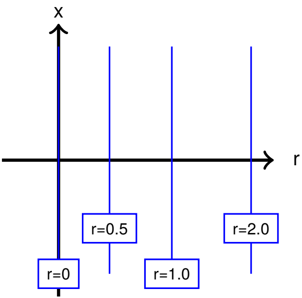

Figure 1 : Constant resistance contours in the Z plane

Bilinear Transformation

$$\Gamma = \frac{z-1}{z+1}$$

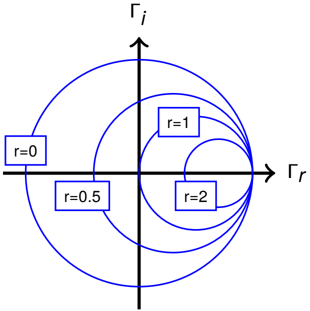

Figure 2 : Constant resistance contours in the Γ plane

Circles of constant reactance

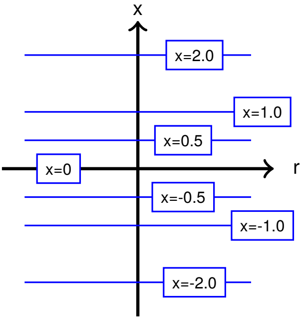

Figure 3 : Constant resistance contours in the Z plane

Bilinear Transformation

$$\Gamma = \frac{z - 1}{z + 1}$$

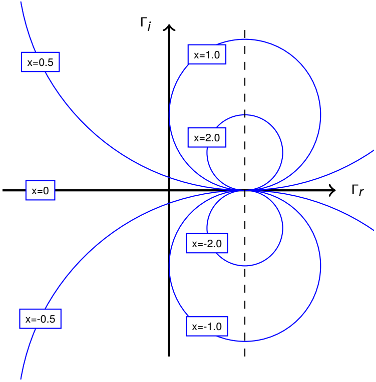

Figure 4 : Constant reactance contours in the Γ plane

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Building the complete Smith Chart

We can now proceed to construct the complete Smith Chart, by combining a set of constant resistance circles and another set of constant reactance circles together on the same Γ plane axis, as illustrated in figure opposite.

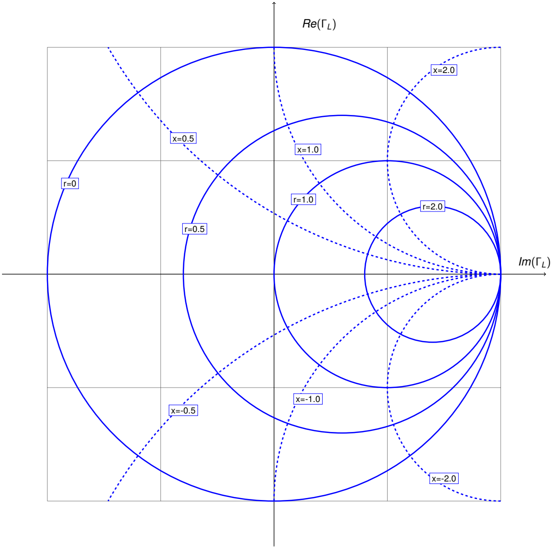

Figure 5 : Combined constant resistance and reactance contours in the Γ plane

Complete Smith Chart

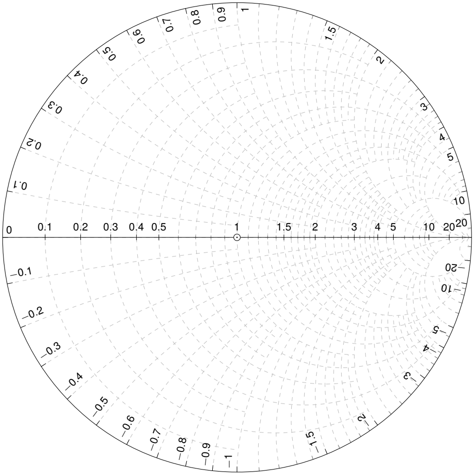

Figure 6 : Smith Chart

Smith Chart regions

1. The upper half of the chart, above the horizontal axis, represents all impedances with a positive reactive part (i.e. inductive impedances).
2. The lower half of the chart represents all impedances with a negative reactive part (i.e. capacitive impedances).
3. The horizontal axis represents all pure resistances.
4. The outer perimeter of the Smith Chart represents all purely reactive impedances (i.e. zero resistance). These are pure inductances on the upper semicircle and pure capacitances on the lower semicircle.
5. The rightmost point on the horizontal axis represents infinite impedance (a perfect open circuit).
6. The leftmost point on the horizontal axis represents zero impedance (a perfect short circuit).

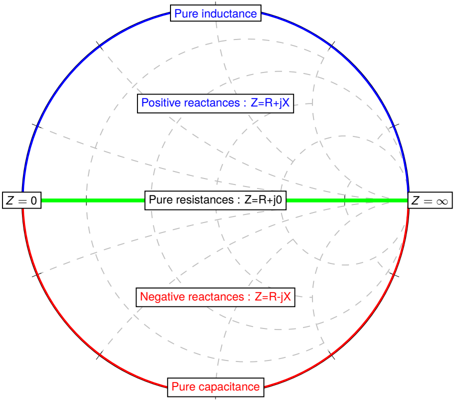

Figure 7 : Smith Chart regions

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Conversion between immittance and reflection coefficient

Consider the load impedance ($Z_L = 25 - j50\Omega$.) which can be normalised as follows:

$$z_L = \frac{(25 - j50)}{50} = 0.5 - j$$

The point y_L represents the normalised admittance of the load. We can determine its value by reading off the constant conductance and constant susceptance circles that this point lies on. This gives us a normalised admittance of :

$$y_L = 0.4 + j0.8$$

We can denormalise this to get the ohmic load admittance Y_L as :

$$Y_L = \frac{(0.4 + j0.8)}{50} = 0.008 + j0.016 \ siemens$$

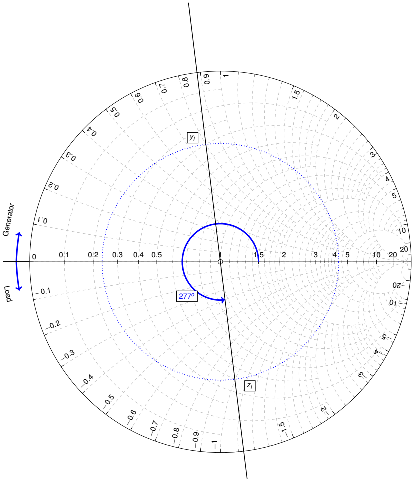

Figure 8 : Immitance / reflection coefficient conversion

Impedance at any point on a transmission line

- Circles of various radii on the Smith Chart, with centres at the origin, represent a constant SWR, which is equivalent to a constant magnitude of reflection coefficient.
- Figure 9 illustrates this for VSWR=3.0, VSWR=8.0 and VSWR=1.5.
- The radius of any such circle determines the magnitude of the reflection coefficient, which lies in the range 0 to 1 in the case of a standard (not compressed) Smith Chart. This encompasses all possible passive values of impedance (or admittance).
- Any point on one of these circles, therefore, represents a point on a lossless transmission line at some distance from the load, since, as we travel away from the load on a lossless line, the reflection coefficient magnitude remains constant but the angle of the reflection coefficient changes.

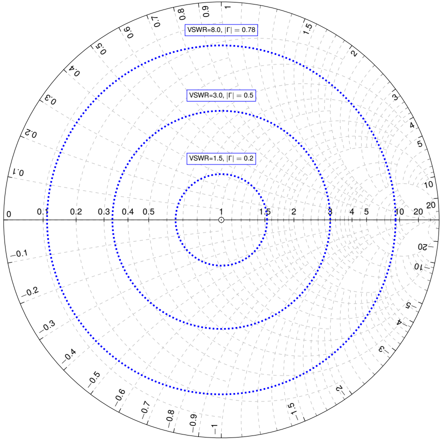

Figure 9 : Constant VSWR circles

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Constant Q contours on the Smith Chart

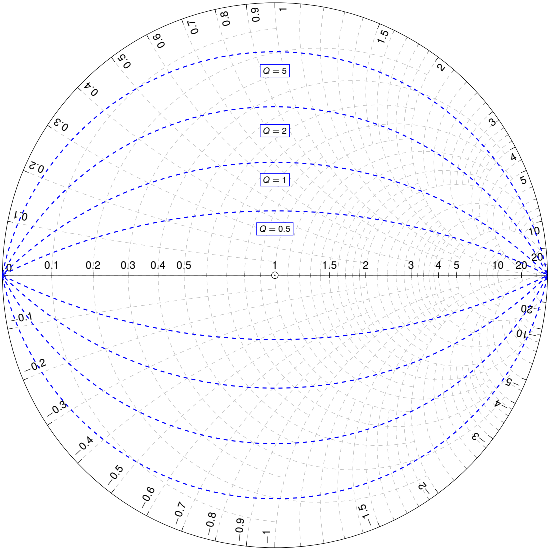

Figure 10 : Constant Q contours on the Smith Chart

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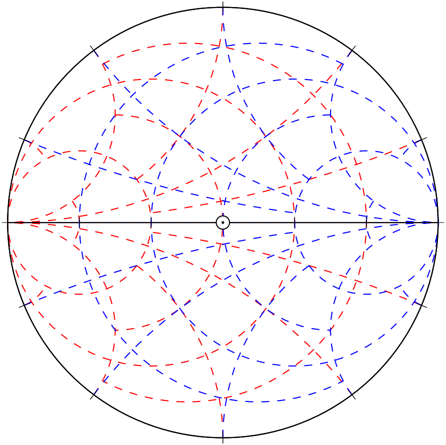
Combined Smith Chart

- That is to say, a mapping of admittances onto the reflection coefficient plane can be obtained by rotating the mapping of impedances onto the reflection coefficient plane (i.e. the conventional Smith chart) by 180^o.
- It is sometimes convenient, although somewhat cluttered, to display both of these Smith charts simultaneously, as shown in figure **??**.



Compressed Smith Chart

- In some cases, where active microwave devices are involved, we need to represent values of negative resistance on the Smith Chart, which implies a reflection coefficient magnitude greater than unity.
- Negative resistance is simply a way of representing the fact that the power wave reflected from a given termination is greater in magnitude than the incident power wave.

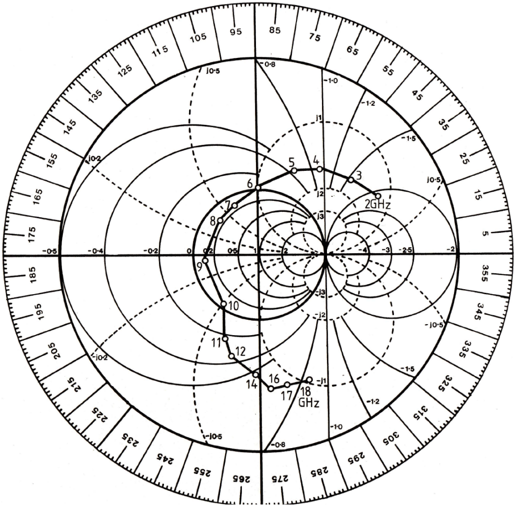


Figure 11 : Compressed Smith Chart used to plot negative resistance behaviour of a MHG9000 MESFET

References

P H Smith.
Electronic Applications of the Smith Chart.
Noble Publishing Corporation, October 2000.